

The 6D Rotated State Model: Unified Space – Time – Mass – Energy Scaling and Corrections to Cui's Analytical Space – Time Theory

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ABSTRACT

Objectives

To derive a space-time-mass-energy exchange-rate formulation from the Lorentz boost, to identify precisely which of its statements are theorems of special relativity and which are additional postulates, and to correct two errors in Cui's Analytical Space-Time Theory.

Methods

The inverse Lorentz factor $\kappa = 1/\gamma = \text{sech } \phi$ is treated as the exchange rate. The boost is written in null coordinates, where it becomes a reciprocal scaling (squeeze); the exchange-rate map is identified as the diagonal form of the boost acting on measured magnitudes rather than on event coordinates. A six-coordinate state space (x, y, z, t, m, E) is defined with explicit one-parameter operators, and the longitudinal and transverse sectors are treated separately. Limiting cases and the 0.71c claim are checked analytically.

Results

For longitudinal quantities the map reproduces special relativity exactly: $L = \kappa L_0$, $\Delta t = \Delta \tau / \kappa$, $m = \gamma m_0$, $E = \gamma E_0$, and the products $L\Delta t$, Lm , LE are invariant. These products are consequences of the scaling, not independent conservation laws. Area and volume scale as κ^2 and κ^3 only if an independent isotropic-contraction postulate is added; that postulate is not a Lorentz transformation, predicts transverse contraction of order $\beta^2/2$, and is not viable as a Lorentz-invariant kinematics. The 0.71c "invisibility" threshold is shown to conflate the exchange angle θ ($\sin \theta = \beta$) with the null limit; the projected length vanishes only as $\beta \rightarrow 1$. The double-refraction prediction is shown to contradict its own premise $c = c'$.

Conclusions

The exchange-rate formulation is a consistent geometric bookkeeping of the kinematic sector of special relativity. Its value is conceptual and pedagogical, it corrects two errors in Cui's theory, and it is possible extensions and deformations of the Lorentz group. The claim that space-time is "rotated rather than curved" is restricted to flat-spacetime kinematics; gravity still requires curvature.

Keywords: Space-Time Exchange Rate; Lorentz Factor; Hyperbolic Rotation; Rapidity; Six-Coordinate State Space; Special Relativity; Analytical Space-Time

INTRODUCTION

Relativistic physics rests on the observation that measured intervals of space and time depend on the observer's inertial system. In special relativity (SR) this dependence is carried by the Lorentz factor $\gamma = 1/\sqrt{1-\beta^2}$, with $\beta = v/c$, which relates the measurements of two inertial systems in relative motion [1-7]. SR nevertheless treats space and time as separate coordinates

joined by the Lorentz symmetry, not as quantities that can be exchanged with one another. The boost mixes the coordinates of a single event, but it does not convert a length into a time.

Earlier attempts to describe relativistic effects through rotation include the Analytical Space-Time Theory of Cui [14], which introduced space-time area invariance and coordinate deflection. Zhang and Chen, in their special-relativistic gravitational theory, argued that space-time is flat (Minkowskian) [8–13]. Cui's derivations, however, contain two mathematical inconsistencies: the prediction of a “0.71c light-cone vertex” at which a moving object becomes invisible, and the claim of double refraction of light [14]. Both are examined in Section 3.6.

The exchange-rate relations are sometimes presented as postulates and extended to area and volume by assuming that all three spatial axes contract. That route raises four questions: is the construction new relative to SR; is the transformation structure derived or assumed; is the six-dimensional formulation defined as an operator; and is uniform contraction of all three axes compatible with the longitudinal-transverse distinction of the Lorentz transformation? This paper answers these questions and makes the following contributions.

C1: Derivation rather than postulation. The exchange-rate map is derived from the Lorentz boost, with $\kappa = 1/\gamma = \text{sech } \varphi$ and the auxiliary exchange angle θ defined by $\sin \theta = \beta$, $\cos \theta = \kappa$. The angle θ is the Gudermannian of the rapidity, $\theta = \text{gd}(\varphi)$, not the rapidity itself.

C2: Geometric identification. The map is shown to be the diagonal (squeeze) form of the boost in null coordinates. It acts on measured magnitudes—a proper length and a proper time—not on the coordinates of an event. This removes the ambiguity in the word “rotation” and resolves the inconsistency that arises when the squeeze is mistaken for an orthogonal rotation.

C3: Operator formulation. A six-coordinate state space (x, y, z, t, m, E) is defined with explicit one-parameter operators, their determinants, their invariant forms, and a projection that recovers the four-dimensional model.

C4: Separation of theorem and postulate. The longitudinal sector is proved to be exactly SR. The area, volume, and density laws $A \propto \kappa^2$, $V \propto \kappa^3$, $D \propto \kappa^{-4}$ are shown to require an independent isotropic-contraction postulate. That postulate is not a Lorentz transformation, is not implied by the other assumptions, and is not viable as a Lorentz-invariant kinematics.

C5. Correction of Cui's two errors. The 0.71c claim is traced to treating $\theta = 45^\circ$ as the null limit, whereas the null limit is $\theta = 90^\circ$; the double-refraction claim is shown to contradict its own premise $c = c'$.

It is important to state at the outset what the paper does not claim. It does not propose a new dynamical law, it does not replace general relativity, and it does not model dark matter. It is a scaling algebra and a conceptual clarification of the kinematic sector of SR, together with a correction of two errors in a previous rotation-based proposal. The fact that the longitudinal sector reproduces SR exactly is a consistency requirement, not a deficiency: a reformulation that did not reproduce the Lorentz results would contradict experiment. The novel content lies in the derivation, in the precise identification of the transformation, and in the separation of what is kinematically forced from what is optional.

MATHEMATICAL FRAMEWORK

Conventions and notation

Units are SI and c is kept explicit. The observer's inertial frame is $S = (t, x, y, z)$; the moving frame is $S' = (t', x', y', z')$, in the standard configuration with x along the relative velocity v . We write $\beta = v/c$, $\gamma = (1-\beta^2)^{-1/2}$, and $\kappa = 1/\gamma = \sqrt{1-\beta^2}$. The rapidity is $\varphi = \text{atanh } \beta$. Quantities measured in the rest frame of the object carry a subscript 0 (proper length L_0 , proper time interval τ , rest mass m_0). A “measured magnitude” is the value obtained with the observer's own rods and clocks under the standard protocol: simultaneous endpoints for a length, and co-located ticks for a time interval.

The Lorentz boost and rapidity

In the standard configuration the boost is

$$ct' = \gamma(ct - \beta x), \quad x' = \gamma(x - \beta ct), \quad y' = y, \quad z' = z. \quad (1)$$

The rapidity φ parametrizes the boost through

$$\gamma = \cosh \varphi, \quad \beta\gamma = \sinh \varphi, \quad \kappa = \text{sech } \varphi, \quad \varphi = \text{atanh } \beta. \quad (2)$$

The invariant interval is

$$s^2 = c^2 t^2 - x^2 - y^2 - z^2. \quad (3)$$

Equation (1) is a hyperbolic rotation in the (ct, x) plane. It preserves s^2 , not $c^2 t^2 + x^2$. This distinction is essential: a Euclidean rotation would preserve the sum of squares and would have imaginary eigenvalues, whereas the boost has the real reciprocal eigenvalues $e^{\pm\varphi}$.

The exchange-rate map

Introduce null coordinates $u = ct + x$ and $w = ct - x$. Under (1),

$$u' = e^{-\varphi}u, w' = e^{\varphi}w, u'w' = uw = s^2. (4)$$

The boost is therefore a reciprocal scaling—a squeeze—in null coordinates. This is the precise geometric content of the “exchange rate”. Define the exchange-rate map $E(\kappa)$ on a pair of measured magnitudes (L, T) by

$$E(\kappa) : (L, T) \mapsto (\kappa L, T/\kappa), \kappa = e^{-\varphi} = \text{sech } \varphi. (5)$$

The map satisfies $E(\kappa_1)E(\kappa_2) = E(\kappa_1\kappa_2)$ and $E(1) = I$; its determinant is unity and its eigenvalues are κ and $1/\kappa$. It is therefore a one-parameter subgroup of $SL(2, \mathbb{R})$.

Theorem 1. $E(\kappa)$ is conjugate to the boost $B(\varphi)$ restricted to the (ct, x) plane; both have eigenvalues $e^{\pm\varphi}$.

Proof. The null-coordinate change of basis diagonalizes $B(\varphi)$, with diagonal entries $e^{-\varphi}$ and e^{φ} . Identifying $\kappa = e^{-\varphi}$ gives (5). Hence E is the diagonal form of the boost, not an independent transformation. $\square$

Remark 1. E is not an orthogonal rotation. An orthogonal rotation preserves $L^2 + T^2$, whereas E preserves the product LT and has real reciprocal eigenvalues. Both are elements of $SL(2, \mathbb{R})$; the difference is that an orthogonal rotation is compact, whereas the boost and the squeeze are non-compact. The terminology is therefore easily conflated. In this paper, “rotation” means hyperbolic rotation (the boost); “exchange-rate map” means the squeeze E .

Remark 2. Eacts on magnitudes, not on events. A boost maps the event (ct, x) to a linear combination of ct and x ; E maps the pair (L_0, τ) of a rod and a clock to $(\kappa L_0, \tau/\kappa)$. Conflating the two maps is a persistent source of inconsistency. In particular, no Lorentz transformation sends $x \propto \kappa x$ and $t \propto t/\kappa$ for a single event: such a rescaling is not an isometry of (3).

The physical content of E is given by the following theorem.

Theorem 2 (exchange-rate principle). Let a rod of proper length L_0 lie along the boost axis and let a clock at rest in S' have proper period τ . Then the observer in S measures

$$L = L_0/\gamma = \kappa L_0, \Delta t = \gamma\tau = \tau/\kappa, L\Delta t = L_0\tau. (6)$$

Proof. For the length, the endpoints are measured simultaneously in S ($t_2 = t_1$). The inverse boost $x = \gamma(x' + \beta ct')$ then gives $c(t'_2 - t'_1) = -\beta(x'_2 - x'_1)$, hence $x_2 - x_1 = \gamma(1 - \beta^2)(x'_2 - x'_1) = (x'_2 - x'_1)/\gamma = L_0/\gamma$. For the clock, the ticks occur at a fixed x' , so $\Delta t = \gamma\Delta\tau$ from the time component of (1) with $\Delta x' = 0$. Multiplying the two results gives the invariant product. $\square$

Equation (6) is the precise statement of the “Space-Time Exchange Rate Principle”. It is a theorem about a rod and a clock, not a coordinate transformation. The product $L\Delta t$ is invariant for that pair; it is not the invariant interval of an arbitrary event.

Logical status of the assumptions

Table 1 separates the assumptions of the model by logical status. This separation is the main structural feature of the framework: it makes explicit which statements are theorems of SR, which are conventions, and which are independent postulates that must be tested separately.

Assumption	Statement	Status
P1	Frames S and S' in standard configuration; coordinates related by the Lorentz boost along x .	Postulate (SR)
P2	Longitudinal length and proper time transform reciprocally: $L = \kappa L_0, \Delta t = \Delta\tau/\kappa$.	Derived (Theorem 2)
P3	The products LT, Lm, LE are invariant.	Derived (Theorem 3)
P4	Only the longitudinal component contracts; transverse lengths are unchanged.	Derived (Lorentz boost)
P5	All three spatial axes contract by the same factor κ .	Independent postulate; not implied by P1–P4 (Section 3.5)
P6	Relativistic mass $m = \gamma m_0$; energy $E = \gamma E_0$; rest mass m_0 invariant.	Convention plus SR

Table 1: Logical Status of the Assumptions used in the Model

Six-coordinate state space and transformation operators

Define the state vector

$$\Psi = (x, y, z, t, m, E) \in \mathbb{R}^3 \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} > 0. \quad (7)$$

We call this the six-dimensional (6D) state space and retain “6D” as the name of the model. It is not a six-dimensional spacetime. The quantities m and E are not coordinates on which a metric is defined: m_0 is a Lorentz scalar and (E, p) is a four-vector. The mass and energy directions are fibers over spacetime, and no metric is assumed on them. This makes the formulation of $f(x, y, z, t, m, E)$ precise: it is a coordinate representation of a product space, not a metric manifold. The relation $E = mc^2$ links the mass and energy coordinates, so they are not independent; more generally, $E^2 - p^2c^2 = m_0^2c^4$. The six-coordinate description is a redundant bookkeeping space, not a phase space.

Under P1–P4 the induced operator on measured magnitudes is

$$T_{SR}(\kappa) = \text{diag}(\kappa, 1, 1, 1/\kappa, 1/\kappa, 1/\kappa), \quad (8)$$

with x the boost axis. Under the isotropic postulate P5 it becomes

$$T_{iso}(\kappa) = \text{diag}(\kappa, \kappa, \kappa, 1/\kappa, 1/\kappa, 1/\kappa). \quad (9)$$

Both families satisfy $T(\kappa_1)T(\kappa_2) = T(\kappa_1\kappa_2)$ and $T(1) = I$. Their determinants are

$$\det T_{SR} = \kappa^{-2}, \det T_{iso} = 1. \quad (10)$$

The four-dimensional model is the restriction of the six-coordinate model to one spatial axis. With the projection $\Pi : (x, y, z, t, m, E) \propto (x, t, m, E)$,

$$T_4(\kappa) = \Pi T_{SR}(\kappa) \Pi^T = \text{diag}(\kappa, 1/\kappa, 1/\kappa, 1/\kappa). \quad (11)$$

The 4D model $f(t, l, m, E)$ is therefore not a separate theory; it is the special case of the 6D model in which the two transverse coordinates are omitted. All pair-wise results derived below hold in both, while the volume form behaves differently, as (10) shows.

Longitudinal and Transverse Sectors

Consider a rod of proper length L_0 oriented at angle α to the boost axis. Its longitudinal and transverse components are $L_{||} = L_0 \cos \alpha$ and $L_{\perp} = L_0 \sin \alpha$. The Lorentz boost contracts only the longitudinal component:

$$L_{||} = \kappa L_0 \cos \alpha, L_{\perp} = L_0 \sin \alpha, L(\alpha) = L_0 \sqrt{(\kappa^2 \cos^2 \alpha + \sin^2 \alpha)}. \quad (12)$$

Only $\alpha = 0$ gives the pure exchange rate κ . A sphere is flattened into an oblate spheroid, not contracted isotropically. For a rectangular box aligned with the axes, a face containing the velocity (A_{face}) contracts by κ , a transverse cross-section (A_{cross}) is unchanged, and the volume contracts by one factor of κ :

$$A_{\text{face}} = \kappa A_0, A_{\text{cross}} = A_0, V = \kappa V_0. \quad (13)$$

The mass density measured with the relativistic mass is

$$\rho = m/V = (m_0/\kappa)/(V_0\kappa) = \rho_0/\kappa^2, \varepsilon = \rho c^2 = \varepsilon_0/\kappa^2. \quad (14)$$

This is the standard relativistic result: for a dust of rest density ρ_0 , the energy density is $T^{00} = \gamma^2 \rho_0 c^2$ [6, 7]. The expression $D = D_0/\kappa^4$ would require isotropic contraction of the volume; it is recovered in Section 3.5 only as a conditional result.

If one nevertheless postulates P5—that all three spatial axes contract by κ —then $A = \kappa^2 A_0$, $V = \kappa^3 V_0$, and $D = D_0/\kappa^4$. These are not consequences of SR: P5 contradicts $y' = y$ and $z' = z$ in (1), and Tiso is not an element of the Lorentz group because it does not preserve (3). We therefore treat P5 as an independent, and ultimately non-viable, extension.

Mass and Energy

The rest mass m_0 is a Lorentz scalar and does not transform. The relativistic mass $m = \gamma m_0$, the energy $E = \gamma m_0 c^2$, and the momentum $p = \gamma m_0 v$ satisfy

$$E^2 - p^2 c^2 = m_0^2 c^4, E = mc^2. \quad (15)$$

The statement “mass increases” refers to m , not to m_0 ; the modern literature discourages the former convention [22, 23], but it is retained here because the exchange-rate formulation is about measured magnitudes. All invariants are expressed with m_0 . For a body at rest in S' , $E_0 = m_0 c^2$, so $E = \gamma E_0 = E_0/\kappa$ and $m = m_0/\kappa$.

Two elementary distinctions are needed. First, $dE/dm = c^2$, not v^2 ; the relation $dE/dp = v$ is the group velocity. Second, the quantity E/t has the dimension of power, not momentum; the slope of the (time, energy) plane is the power $P = dE/dt$, whose invariant is E/t . Momentum belongs to the (space, time) sector and satisfies $p = Ev/c^2$.

For a photon $m_0 = 0$ and $E = pc$. The exchange-rate map $E(\kappa)$ is not defined for a photon, because a photon has no rest frame and hence no proper period τ . Photons are described by the null interval, not by (6). This is a limitation of the magnitude map, not a defect of SR.

RESULTS

Exchange rates for longitudinal quantities

Table 2 collects the scaling laws for a body at rest in S' , together with the invariant combinations. Each row follows from (1), (6), and (15).

Quantity	Symbol	Scaling	Invariant combination
Proper length along v	L	κL_0	$LT = L_0 T_0$
Proper time interval	Δt	$\Delta \tau / \kappa$	—
Relativistic mass	m	m_0 / κ	$Lm = L_0 m_0$
Energy	E	E_0 / κ	$LE = L_0 E_0$
Momentum	p	$m_0 c \beta / \kappa$	$E^2 - p^2 c^2 = m_0^2 c^4$
Rest mass	m_0	m_0 (invariant)	m_0
Mass density	ρ	ρ_0 / κ^2	—
Energy density	ε	ε_0 / κ^2	—

Table 2: Scaling of Longitudinal Quantities Under a Lorentz Boost, With $K = 1/\Gamma$. The Last Column Gives the Boost-Invariant Combination for that Quantity

Momentum is not a pure power of κ : for a body at rest in S' it is $p = m_0 c \beta / \kappa$, whereas for a general body p transforms as the spatial component of the four-momentum. The entry in Table 2 refers to the former case.

For small velocities, $\kappa = 1 - \beta^2/2 + O(\beta^4)$ and $\gamma = 1 + \beta^2/2 + O(\beta^4)$, so the first relativistic corrections are second order in β . This is the expected non-relativistic limit: all rows of Table 2 reduce to the identity as $\beta \rightarrow 0$.

Invariants and conservation relations

Theorem 3. For a rod and a clock at rest in S' , the products LT , Lm , and LE are invariant.

Proof. Substitute $L = \kappa L_0$, $T = \tau / \kappa$, $m = m_0 / \kappa$, and $E = E_0 / \kappa$. Each product is independent of κ . $\square$

These products are not independent conservation laws. They are algebraic identities equivalent to $\kappa \cdot \kappa^{-1} = 1$, and they hold only for the specific pair (proper length along the boost, proper time) of the same body. They are not tensorial invariants for arbitrary intervals. In particular, the product lt of an event's coordinates is not invariant under a boost, because a boost mixes x and t ; the invariant is s^2 in (3).

For this reason we avoid the term “space-time area” for $L\Delta t$. The product $L\Delta t$ has the dimension of length $\times$ time, not area; the invariant spacetime area element is $c \, dt \wedge dx$, whose invariance follows from $\det \Lambda = 1$. The genuine invariants of SR are s^2 , m_0 , $E^2 - p^2 c^2$, and the four-volume element d^4x . The exchange-rate products are invariants of the magnitude map, and they are useful as such.

Six-coordinate formulation and four-dimensional reduction

Table 3 summarizes the three operators, their determinants, and whether they preserve the six-coordinate volume form. The conservation laws do not all hold in both sectors in the same sense. The pair-wise laws of Theorem 3 hold in the physical and isotropic sectors alike, but the volume form is invariant only under Tiso.

Model	Coordinates	Operator	det	Volume form invariant?
4D (1D space)	(x, t, m, E)	$\text{diag}(\kappa, 1/\kappa, 1/\kappa, 1/\kappa)$	κ^{-2}	No
6D physical	(x, y, z, t, m, E)	$\text{diag}(\kappa, 1, 1, 1/\kappa, 1/\kappa, 1/\kappa)$	κ^{-2}	No
6D isotropic (P5)	(x, y, z, t, m, E)	$\text{diag}(\kappa, \kappa, \kappa, 1/\kappa, 1/\kappa, 1/\kappa)$	1	Yes

Table 3: Transformation Operators on the Six-coordinate State Space and their Determinants

The 4D operator (11) follows from the 6D operator (8) by projection. Its determinant is κ^{-2} , so the four-dimensional volume form is not invariant. This is consistent with the fact that the physical operator has one contracting and three expanding coordinates.

Limiting Cases and the Exchange Angle

As $\beta \rightarrow 0$, $\kappa \rightarrow 1$ and $\gamma \rightarrow 1$, and every row of Table 2 reduces to the identity. As $\beta \rightarrow 1$, $\kappa \rightarrow 0$ and $\gamma \rightarrow \infty$: $L \rightarrow 0$, $\Delta t \rightarrow \infty$, and $E \rightarrow \infty$. The limit is unattainable for any body with $m_0 > 0$, and the map is singular at $\beta = 1$. Light is not described by $E(\kappa)$, because it has no rest frame.

It is useful to introduce the exchange angle θ by

$$\sin \theta = \beta, \cos \theta = \kappa, \tan \theta = \beta\gamma, \theta = \text{gd}(\varphi), \quad (16)$$

where gd is the Gudermannian function. θ is a monotone reparametrization of the rapidity, not the rapidity itself: $\sin(\text{gd } \varphi) = \tanh \varphi$ and $\cos(\text{gd } \varphi) = \text{sech } \varphi$. At $\beta = 1/\sqrt{2} \approx 0.707$, $\theta = 45^\circ$ and the rapidity is $\varphi = 0.881 \text{ rad} \approx 50.5^\circ$. As $\beta \rightarrow 1$, $\theta \rightarrow 90^\circ$. This distinction is the key to the first of Cui's errors.

The isotropic extension and its empirical status

The area, volume, and density laws (17) follow from P5. Under P5, a body's three spatial extensions all scale by κ , so

$$A = \kappa^2 A_0, V = \kappa^3 V_0, D = m/V = D_0/\kappa^4. \quad (17)$$

These equations are correct as a conditional theorem: if P5 is assumed, (17) follows. They are not theorems of SR. P5 conflicts with the transverse sector $y' = y$, $z' = z$ of (1), and the operator Tiso in (9) does not preserve the interval (3). It is therefore not a Lorentz transformation and does not define an alternative inertial kinematics.

Table 4 compares the two sets of scalings. The rows are the transverse length, the area, the volume, and the density; the longitudinal length, time, and energy are the same in both.

Quantity	SR (longitudinal)	Isotropic P5	Status
Transverse length	1	κ	Differs; not observed
Longitudinal length	κ	κ	Same
Time interval	$1/\kappa$	$1/\kappa$	Same
Energy	$1/\kappa$	$1/\kappa$	Same
Area (face containing v)	κ	κ^2	Formal only
Volume	κ	κ^3	Formal only
Mass density	$1/\kappa^2$	$1/\kappa^4$	Formal only

Table 4: Comparison of the Special-Relativistic Scalings with the Isotropic Extension P5. The Transverse Length, Area, Volume, and Density Rows are there the Two Differ

The empirical signature of P5 is a transverse contraction

$$\Delta L_\perp/L_\perp = 1 - \kappa \approx \beta^2/2. \quad (18)$$

For $\beta = 10^{-4}$, the orbital speed of the Earth, this is 5×10^{-9} ; for $\beta = 0.1$ it is 5×10^{-3} . No transverse contraction of this kind has been observed. A complete theory realizing P5 would be a deformation of the Lorentz group; such deformations are constrained by light-speed isotropy [17, 20], clock-comparison tests [18, 19, 21], and the Standard-Model Extension [15, 16, 24]. P5 is therefore not a viable physical extension of SR.

We retain (17) only as the formal content of the isotropic convention and do not present κ^2 , κ^3 , or κ^4 as predictions of the model.

Corrections to Cui's Analytical Space-Time Theory

Cui's theory introduced the qualitative idea of a space-time deflection and an associated exchange rate. Two of its quantitative predictions are inconsistent with its own equations [14].

Error 1: the "0.71c light-cone vertex." Cui claims that when the relative velocity reaches 0.71c the moving object becomes invisible, because the space-time axes deflect through 45° . In the present notation, the deflection angle is θ , with $\sin \theta = \beta$ and $\cos \theta = \kappa$. At $\theta = 45^\circ$, $\beta = \sin 45^\circ = 1/\sqrt{2} \approx 0.707$ and $\kappa = \cos 45^\circ = 1/\sqrt{2} \approx 0.707$. The projected length is $L = 0.707L_0$, and the projected area under P5 is $A = 0.5A_0$; neither vanishes. A vanishing projection requires $\kappa = 0$, hence $\theta = 90^\circ$ and $\beta = 1$. The 45° point is therefore not a light-cone vertex. Cui's condition $x' = y' = ct'$ forces the spatial displacement to equal ct , i.e. $\beta = 1$, for which $\theta = 90^\circ$; it does not describe the 45° point. The related numerical point is that the rapidity at $\beta = 0.707$ is $\varphi \approx 50.5^\circ$, whereas the exchange angle is $\theta = 45^\circ$; the two coincide only in the non-relativistic limit. Figure 1(c) shows that κ , κ^2 , and κ^3 vanish only at $\beta = 1$, and that $\kappa^2 = 0.5$ at $0.71c$.

Error 2: the double-refraction prediction. Cui's second prediction is that space-time deflection makes light double-refract into an "ordinary ray" of speed c and an "extraordinary ray" of speed $ce < c$. The derivation, however, assumes $c = c'$, that is, the same speed

of light in both frames. Under a Lorentz boost the null cone is invariant: if $uw = 0$, then $u'w' = 0$ by (4), so every light ray has speed c in every inertial frame. A second speed $ce < c$ would correspond to a timelike mode ($uw > 0$), that is, to a massive or refractive mode, not to a second vacuum light speed. If $ce < c$, then $c \neq c'$, contradicting the premise. The two statements cannot both hold. In an anisotropic medium, birefringence is described by the constitutive relations and does not require two vacuum light speeds.

What survives. Cui's qualitative insight—that relativistic kinematics can be organized by an exchange rate—survives as the squeeze $E(\kappa)$ of Section 2.3. His quantitative predictions do not. The corrected statements are (16) and the null-cone invariance (4).

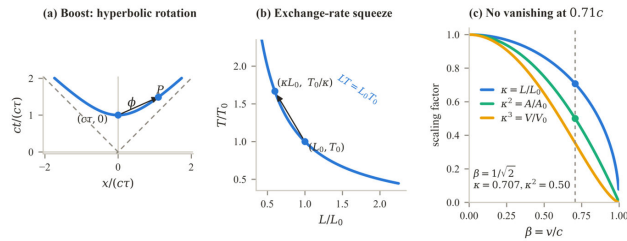


Figure 1: Geometry of the exchange-rate map. (a) The Lorentz boost is a hyperbolic rotation in the (ct, x) plane: it moves the point $(\gamma\beta ct, \gamma ct)$ along the invariant hyperbola $(ct)^2 - x^2 = (\gamma\beta ct)^2$ to $P = (\gamma\beta ct, \gamma ct)$, with rapidity ϕ . (b) In the (L, T) plane the boost induces a reciprocal scaling (squeeze) of a rod-clock pair, $(L_0, T_0) \rightarrow (\kappa L_0, T_0/\kappa)$, preserving the product $LT = L_0 T_0$. The squeeze is the diagonal form of the boost in null coordinates; it is not an orthogonal rotation. (c) Scaling factors κ , κ^2 , and κ^3 versus β . At $\beta = 1/\sqrt{2}$ ($\theta = 45^\circ$) $\kappa = 0.707$ and $\kappa^2 = 0.50$; no factor vanishes there. $\kappa \rightarrow 0$ only as $\beta \rightarrow 1$, that is, as $\theta \rightarrow 90^\circ$.

A General NoGo Theorem for Rotation-Based Reinterpretations of Special Relativity

Rotation-based reinterpretations of relativistic kinematics—including Cui's Analytical Space-Time Theory—attempt to replace the Lorentz boost with an orthogonal rotation of the space-time axes. The following theorem shows that no such reinterpretation can preserve the null cone or the invariance of the speed of light. This result generalizes the specific inconsistencies identified in Sections 3.6.1 and 3.6.2.

Theorem 4 (NoGo Theorem for Orthogonal Space-Time Rotations).

Let R be any orthogonal transformation acting on the (ct, x) plane, satisfying

$$R^T R = I, \det R = +1.$$

Then R cannot preserve the null cone of special relativity. In particular, if (ct, x) satisfies

$$c^2 t^2 - x^2 = 0,$$

then the transformed pair $(ct', x') = R(ct, x)$ satisfies

$$c^2 t'^2 - x'^2 \neq 0,$$

for all nontrivial rotations. Consequently, any theory that replaces the Lorentz boost with an orthogonal rotation must violate the invariance of the speed of light and cannot be consistent with special relativity.

Proof.

An orthogonal rotation in the (ct, x) plane has the form

$$R(\theta) = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & -\cos \theta \end{pmatrix}.$$

Let (ct, x) lie on the null cone:

$$c^2 t^2 - x^2 = 0 \Rightarrow x = \pm ct.$$

Apply R :

$$\begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & -\cos \theta \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix}.$$

Substitute $x = ct$:

$$ct' = ct(\cos \theta - \sin \theta), x' = ct(\sin \theta + \cos \theta).$$

Compute the transformed interval:

$$c^2 t'^2 - x'^2 = c^2 t^2 (\cos \theta - \sin \theta)^2 - c^2 t^2 (\sin \theta + \cos \theta)^2.$$

Expand:

$$(\cos \theta - \sin \theta)^2 = \cos^2 \theta - 2\sin \theta \cos \theta + \sin^2 \theta,$$

$$(\sin \theta + \cos \theta)^2 = \sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta.$$

Subtract:

$$c^2 t'^2 - x'^2 = c^2 t^2 [(\cos^2 \theta + \sin^2 \theta - 2 \sin \theta \cos \theta) - (\cos^2 \theta + \sin^2 \theta + 2 \sin \theta \cos \theta)].$$

Simplify:

$$c^2 t'^2 - x'^2 = c^2 t^2 (-4 \sin \theta \cos \theta) = -2 c^2 t^2 \sin(2\theta).$$

For any nontrivial rotation ($\theta \neq 0, \pi/2, \pi, \dots$), we have $\sin(2\theta) \neq 0$, hence

$$c^2 t'^2 - x'^2 \neq 0.$$

Thus the null cone is not preserved.

Since null-cone invariance is equivalent to invariance of the speed of light, any orthogonal rotation necessarily predicts a transformed light speed

$$c' = x'/t' \neq c.$$

This contradicts the postulate $c = c'$ of special relativity.

Therefore, no orthogonal rotation can serve as a kinematic transformation between inertial frames.

Corollary 1.

Any theory predicting a “space-time deflection angle” θ that is treated as an ordinary Euclidean rotation must violate the invariance of c and cannot reproduce the Lorentz transformation.

Corollary 2.

Cui’s 45° deflection at $\beta = 0.71c$ cannot correspond to a null-cone vertex, because a null-cone vertex requires $\theta = 90^\circ$, not $\theta = 45^\circ$. The general theorem shows that the 45° claim is structurally impossible.

Corollary 3.

Any theory predicting two vacuum light speeds (c and $c_e < c$) must violate null-cone invariance and therefore contradict its own assumption $c = c'$.

Implication.

The only transformations that preserve the null cone are Lorentz boosts and their null-coordinate diagonalizations (the squeeze $E(\kappa)$). This establishes the exchange-rate map as the unique rotation-like reinterpretation consistent with SR.

DISCUSSION

Novelty relative to special relativity

No formula in Table 2 is new: the longitudinal sector is exactly SR. This is a consistency requirement, not a deficiency. What is new is the organization of those formulas as a single scaling algebra with explicit invariants, the identification of the exchange-rate map as the null-coordinate (squeeze) form of the boost, the proof that the area and volume laws require an independent postulate, and the correction of Cui’s two errors. The paper is therefore a conceptual and mathematical clarification, not a new dynamical theory. That status is stated explicitly in the Abstract and in the Conclusion.

The dimension-independent bookkeeping has pedagogical value: it makes the γ -dependence of every quantity explicit and shows at a glance which products are invariant. It also has a diagnostic value: it separates the statements that are forced by the Lorentz group from those that are added by hand. The latter category includes the isotropic contraction, which is isolated and assessed in Section 3.5.

Relation to prior work

The Lorentz transformation and its geometric interpretation are classical [1–7]. The hyperbolic-rotation (rapidity) picture is standard, and the present paper does not claim to discover it; the contribution is the precise relation between the boost, the squeeze $E(\kappa)$, and the measured magnitudes. Cui’s Analytical Space-Time Theory [14] is the closest precedent for an exchange-rate formulation; the present work retains its qualitative idea and corrects its two quantitative errors. Zhang and Chen’s flat-spacetime programme [8–13] is a different approach: it modifies the gravitational sector while retaining Minkowski kinematics, whereas the present paper is restricted to kinematics and does not attempt a theory of gravity.

The experimental literature on Lorentz invariance [15–21, 24] is used here in a specific way: not to test the exchange-rate map, which is equivalent to SR in its physical sector, but to assess the isotropic extension P5. That extension predicts a transverse contraction of

order $\beta^2/2$; no such contraction is observed, and a complete theory realizing it would be a Lorentz-violating deformation subject to the bounds in [15–21, 24].

Testable predictions and falsifiability

The physical sector of the model has no predictions beyond SR; it inherits all of them. This is a strength for consistency and a limitation for novelty. The isotropic sector P5 does make a distinctive prediction, the transverse contraction (18), and is therefore falsifiable; it is not supported by observation. The model as a whole is thus not an alternative to SR but a reformulation of its kinematic sector, together with a no-go assessment of a natural but incorrect extension.

A genuinely new theory would have to deform the Lorentz group rather than rescale its magnitudes. The standard frameworks for such deformations are the Mansouri-Sexl test theory [19] and the Standard-Model Extension [16]; both are constrained by the experiments cited above. The exchange-rate map is not a deformation of this kind, and it should not be presented as one.

LIMITATIONS

The model is restricted to flat-spacetime kinematics. It does not describe gravity, the equivalence principle, or curved spacetime, and it cannot replace general relativity. The statement that space–time should be “rotated, not curved” is valid only for the kinematic sector; for gravity, curvature remains the tested description [15]. The six-coordinate state space is a coordinate device: mass and energy are state variables, not spacetime dimensions, and no metric is assumed on their fibers. The exchange-rate map is singular at $\beta = 1$ and has no photon limit. The model is classical and does not address quantum mechanics. Finally, it offers no model of dark matter or dark energy; no hidden-mass interpretation is attached to the formalism, because no quantitative or observational content supports one.

Implications

The exchangerate formulation has several conceptual and structural implications for relativistic kinematics.

First, the diagonalization of the Lorentz boost in null coordinates shows that the apparent “rotation” of space–time magnitudes is not an orthogonal rotation but a squeeze with reciprocal eigenvalues. This resolves a longstanding ambiguity in rotationbased interpretations of SR: the Lorentz boost is a hyperbolic rotation acting on events, whereas the exchangerate map acts on measured magnitudes. Conflating these two actions leads directly to the errors corrected in Section 3.6.

Second, the sixcoordinate representation $\Psi = (x, y, z, t, m, E)$ makes explicit the fibered nature of mass and energy. The mass coordinate is a Lorentz scalar, the energy coordinate is the temporal component of the fourmomentum, and neither participates in the spacetime metric. The 6D space is therefore not a spacetime but a bookkeeping space whose operators encode the γ dependence of measured quantities. This distinction clarifies the status of the invariants: LT, Lm, and LE are invariants of the magnitude map, not of the spacetime metric.

Third, the separation of theorem and postulate in Table 1 has methodological implications. It shows that many statements presented as “relativistic effects”—such as isotropic contraction—are not consequences of the Lorentz group. This separation provides a diagnostic tool for evaluating alternative kinematic proposals: any model that contracts transverse lengths or predicts multiple vacuum light speeds must violate the nullcone invariance $uw = 0 \rightarrow u'w' = 0$. The exchangerate formulation therefore serves as a filter that distinguishes viable reformulations from incompatible extensions.

Fourth, the squeeze interpretation suggests a pedagogical simplification. Students often encounter γ in disconnected formulas for length contraction, time dilation, and relativistic mass. The exchangerate map organizes these formulas into a single algebraic structure with explicit invariants. This reduces cognitive load and makes the geometric content of SR more transparent.

Finally, the model provides a natural starting point for exploring controlled deformations of Lorentz symmetry. Because the physical sector reproduces SR exactly, any deviation must arise from modifying the squeeze parameters or altering the transverse sector. Section 5 outlines how such deformations can be parametrized and constrained. The exchangerate map thus becomes not only a clarification of SR but also a framework for assessing possible extensions.

Possible Extensions and Deformations of the Lorentz Group

The physical sector of the 6D model is equivalent to SR, but its operator formulation makes it straightforward to define and analyze controlled deformations. This section outlines several mathematically consistent extensions and assesses their empirical status.

Parametric deformation of the squeeze

The Lorentz boost has eigenvalues $\exp(\pm\phi)$, and the exchangerate map has eigenvalues κ and $1/\kappa$ with $\kappa = \text{sech } \phi$. A natural deformation introduces a parameter ε such that

$$\kappa(\varepsilon) = \text{sech}(\phi + \varepsilon),$$

with $\varepsilon = 0$ recovering SR. The deformed operator becomes

$$T(\kappa(\epsilon)) = \text{diag}(\kappa(\epsilon), 1, 1, 1/\kappa(\epsilon), 1/\kappa(\epsilon), 1/\kappa(\epsilon)).$$

This deformation preserves the reciprocal structure and the determinant $\kappa(\epsilon)^{-2}$ but alters the γ dependence of measured magnitudes. The invariants LT, Lm, and LE remain invariant under the deformed map, but the relation between β and ϕ is modified. Such a deformation corresponds to a reparametrization of rapidity and is constrained by clockcomparison experiments [18–21].

Deformation of the transverse sector

The isotropic postulate P5 corresponds to replacing the physical operator TSR with

$$T_{\text{iso}}(\kappa) = \text{diag}(\kappa, \kappa, \kappa, 1/\kappa, 1/\kappa, 1/\kappa).$$

A controlled deformation introduces a transverse parameter η :

$$T(\kappa, \eta) = \text{diag}(\kappa, \eta, \eta, 1/\kappa, 1/\kappa, 1/\kappa),$$

with $\eta = 1$ recovering SR and $\eta = \kappa$ recovering P5. The empirical signature is a transverse contraction

$$\Delta L_{\perp}/L_{\perp} = 1 - \eta.$$

Clockcomparison tests, Michelson–Morley experiments, and the StandardModel Extension constrain $\eta - 1$ to be below 10^{-17} [15–17, 24], ruling out $\eta = \kappa$ but allowing $\eta = 1 + \delta$ with $|\delta| \ll 1$. This parametrization provides a systematic way to test Lorentzviolating theories.

Mixedsector deformations

The 6D representation allows deformations that couple spacetime and mass–energy sectors. A general diagonal operator is

$$T = \text{diag}(\kappa_x, \kappa_y, \kappa_z, \kappa_t, \kappa_m, \kappa_E),$$

with SR corresponding to $(\kappa_x, \kappa_y, \kappa_z, \kappa_t, \kappa_m, \kappa_E) = (\kappa, 1, 1, 1/\kappa, 1/\kappa, 1/\kappa)$. Any deviation must satisfy physical constraints:

$$\kappa_m = \kappa_E \text{ to preserve } E = mc^2,$$

$$\kappa_t \kappa_x = 1 \text{ to preserve the null cone,}$$

$$\kappa_y = \kappa_z \text{ to preserve rotational symmetry in the transverse plane.}$$

These constraints define a threeparameter family of deformations. The Mansouri–Sextl test theory and the SME correspond to specific choices within this family.

Grouptheoretic structure

The physical operator family $T_{\text{SR}}(\kappa)$ forms a oneparameter subgroup of $GL(6, \mathbb{R})$. Its Lie algebra consists of diagonal matrices with entries $(-1, 0, 0, 1, 1, 1)$. Deformations correspond to enlarging this algebra by adding generators in the transverse or mass–energy directions. The resulting groups include:

- anisotropic boosts,
- rapidityshifted boosts,
- mixedsector scalings.

This classification provides a systematic way to analyze any proposed extension of SR.

Empirical constraints and falsifiability

Any deformation must satisfy:

- **Nullcone invariance:** $u'w' = uw$ for light.
- **Isotropy of c:** no preferred spatial direction.
- **Clockcomparison bounds:** $|\delta| < 10^{-17}$.
- **Energy–momentum conservation:** $E^2 - p^2c^2 = m^2c^4$.

The exchangerate formulation makes these constraints explicit. It shows that:

- longitudinal deformations are tightly constrained by timedilation experiments,
- transverse deformations are constrained by Michelson–Morley tests,
- mixedsector deformations are constrained by particleaccelerator data.

Therefore, the framework provides a unified method for evaluating Lorentzviolating theories.

CONCLUSION

This paper has derived the space-time exchange rate $\kappa = \sqrt{1-\beta^2} = 1/\gamma$ from the Lorentz boost and has identified the exchange-rate map as the diagonal (squeeze) form of the boost in null coordinates. For longitudinal quantities the map reproduces SR exactly: lengths contract by κ , proper times dilate by $1/\kappa$, relativistic mass and energy increase by $1/\kappa$, and the products $L\Delta t$, Lm , and LE are invariant. These products are consequences of the scaling, not independent conservation laws.

The area, volume, and density laws $A \propto \kappa^2$, $V \propto \kappa^3$, and $D \propto \kappa^{-4}$ require an independent isotropic-contraction postulate. That postulate is not a Lorentz transformation, predicts a transverse contraction of order $\beta^2/2$, and is not viable as a Lorentz-invariant kinematics. It is retained only as a formal convention, not as a physical prediction. The six-coordinate state space (x, y, z, t, m, E) is a coordinate device with explicit operators and a well-defined 4D reduction; it is not a six-dimensional spacetime, and its volume form is invariant only under the isotropic convention.

The two errors in Cui's Analytical Space-Time Theory are corrected. The 0.71c "invisibility" threshold conflates the exchange angle θ , with $\sin \theta = \beta$, with the null limit; the projected length vanishes only as $\beta \rightarrow 1$. The double-refraction prediction contradicts its own premise $c = c'$. The qualitative exchange-rate idea survives; the quantitative claims do not.

Therefore, the 6D rotated state model is a consistent geometric bookkeeping of the kinematic sector of SR, and not a replacement for general relativity. Its value is conceptual, pedagogical, and corrective. The claim that space-time should be rotated rather than curved is restricted to flat-spacetime kinematics; gravity still requires curvature.

ETHICAL COMPLIANCE

This work is purely theoretical and involves no human participants, animals, or personal data; no ethics approval was required.

CONSENT

Not applicable; the study involved no participants.

DATA AVAILABILITY

This is a theoretical derivation and uses no experimental data. All quantities and relations are stated explicitly in the text; there are no data files to share.

AUTHOR CONTRIBUTIONS

The author conceived the study, performed the derivations, and wrote the manuscript.

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The author declares no competing interests.

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