

Exact Travelling - Wave Structures of the Caputo Time-Fractional Kadomtsev - Petviashvili Equation via a Unified Cross-Method Riccati - Extended Auxiliary Framework with Fractional - Memory Analysis

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ABSTRACT

This work extends the concept of travelling waves to the Caputo time-fractional Kadomtsev-Petviashvili equation by unifying cross-method approach with the method of exact solutions. By using the Fractional Complex Transformation method, the partial fractional differential equation is turned into a nonlinear ordinary differential equation, the Riccati and Extended Auxiliary Equation method is applied to solve the ordinary differential equation. This leads to systematically classified travelling-wave solutions, which can be divided into three main regimes: smooth bounded travelling waves of tanh/ch, singular periodic travelling waves of tanh/coth branch, and non-periodic travelling waves of inverse power of tan/cot branches. The relationships between the parameters of the two analytical solutions, and the similarities in the functional structure of the two, are used to compare the two sets of solutions, separating those with related parameterizations from those with different wave regimes. The spatial geometry, regularity, periodicity and the singularity structure of the solutions obtained is further characterized by 3-D surfaces, 2-D cross-sectional profiles and Contour representations. The fractional-order analysis reveals that the parameters β of the Caputo fractional derivative influence the temporal evolution, displacement, propagation and phase of travelling structures, but retain the same functional class of their spatial structures. The main innovation is the framework of the Caputo fractional KP which brings together exact analytical construction, cross-method correspondence, systematic wave-Regime's classification, graphical characterization, and fractional-memory analysis in a unified framework.

Keywords: Caputo Fractional Derivative; Kadomtsev-Petviashvili Equation; Fractional Complex Transformation; Riccati Equation Method; Extended Auxiliary Equation Method; Exact Travelling Waves; Singular Periodic Waves; Fractional Memory

INTRODUCTION

Nonlinear evolution equations are basic to modelling dispersive waves and nonlinear interactions in fluid dynamics, plasma physics, nonlinear optics and systems of this type. The Kadomtsev-Petviashvili (KP) equation is an extension of the Korteweg-de Vries model to multidimensional wave propagation and has a rich travelling wave, soliton, periodic and rational structure. Fractional calculus is a natural extension of classical models to take into consideration memory and hereditary effects [1-5]. The Caputo fractional derivative, in particular, has the property to contain the temporal memory without losing the integer order limit. In this regard, in this paper a Caputo time-fractional KP based framework is developed and the governing FPDE is transformed into a nonlinear ODE by using a fractional travelling-wave transformation. [6-12].

The solutions of the reduced equation are obtained by Riccati and Extended Auxiliary Equation methods. The solutions so obtained are organized into three regimes: (i) hyperbolic (smooth tanh and singular coth solutions), (ii) trigonometric/periodic (singular tan and cot solutions), and (iii) rational (non-periodic inverse-power solutions). Other branches are then compared to the corresponding branches in [3,5,6,9,12,13] in order to distinguish mathematically similar from actually different wave representations. [7,14-15].

Main Contributions

1. Caputo fractional KP framework: Implements temporal memory in KP travelling-wave model.
2. Analytical reduction: Transforms fractional PDE to a nonlinear ODE using a fractional travelling wave transformation.

3. Unified exact solutions: Obtains and categorizes families of travelling waves which are hyperbolic, trigonometric/periodic and rational solutions via Riccati and Extended Auxiliary Equation methods.
4. Unified cross-method analysis: Sets up a correspondence between the parameters of different solution branches and elucidates the mathematical relationships, different wave regimes.
5. Wave-structure characterization: differentiates between smooth, singular, periodic and algebraic regimes by analyzing them from an analytical and graphical perspective using 3-D, 2-D, and Contour representations.
6. Fractional-memory analysis: How the Caputo order modifies the temporal evolution, displacement, propagation and phase without changing the underlying spatial wave class.

Research Gap and Positioning of the Present Study

There are existing studies which have established the classical KP travelling-wave theory, Riccati/tanh-coth method, Extended Auxiliary Equation method and several forms of fractional KP [1,15-18]. Nevertheless, these strands are not yet connected within a Caputo time-fractional KP framework that systematically compares the corresponding analytical branches, relates the parameters of these branches, differentiates the smooth hyperbolic, singular hyperbolic, periodic singular, and rational regimes, and consistently relates the analytical solutions with 3D, 2D, and Contour representations and fractional-memory effects [19-21]. The novelty of the present study does not reside in the novelty of the standard tanh, coth, tan, cot or rational functions themselves, but rather in the fact of having them all derived from a common cross-method framework, having their parameters corresponding, having them all derived from a common systematic framework of wave families, having them characterized graphically and having them all interpreted under the common Caputo memory-based scheme within a common fractional KP framework.

S.No	Year	Authors	Research Focus	Methodology / Approach	Key Contribution	Main Limitation / Gap	DOI
1	2006	I. E. Inan; D. Kaya	Potential KP equation and shallow-water-wave equations	Improved tanh function method	Exact solutions of potential KP and shallow-water-wave equations	Classical integer-order setting; no fractional-KP	10.1016/j.physleta.2006.01.106
2	2006	G. Q. Xu	KP and Jimbo-Miwa equations in (3+1) dimensions	Painlevé analysis, truncated Painlevé expansion, bilinear methods	Soliton and dromion solutions	No fractional derivatives	10.1016/j.chaos.2005.08.089
3	2007	Zhengde Dai; Shaolin Li; Qingyun Dai; Jian Huang	Singular periodic soliton solutions and resonance of KP	Two-soliton and bilinear methods	Periodic soliton solutions and resonance interactions	Classical KP	10.1016/j.chaos.2006.04.028
4	2009	Changfu Liu; Theodore E. Simos; George Psihoyios; Ch. Tsitouras	Periodic solitary-wave solutions of potential KP	Hirota bilinear method and graphical analysis	Exact periodic solitary-wave solutions and propagation	Classical potential KP	10.1063/1.3241573
5	2009	Zhengde Dai; Ying Huang; Xing Sun; Dongli Li; Zhaohui Hu	Singular/non-singular solitary waves of KP with p-power nonlinearity	Auxiliary-equation methods	Exact singular and non-singular solitary waves	Classical generalized KP	10.1016/j.chaos.2007.08.050
6	2009	Ahmet Bekir	Travelling-wave solutions of nonlinear	tanh-coth and Riccati methods	Exact travelling and solitary waves	Not specifically fractional-KP	10.1016/j.chaos.2007.09.029

			evolution equations				
7	2012	Nikolai A. Kudryashov; Kirill E. Shilnikov	Critical analysis of tanh-coth/Riccati methods	Riccati and Weierstrass elliptic functions	Identifies errors/redundancies in reported solutions	Methodological critique	10.1016/j.jksus.2012.06.001
8	2014	Gui-qiong Xu	Extended auxiliary equation method	Auxiliary equation and Jacobi elliptic functions	Elliptic, solitary and trigonometric solutions	NLS rather than KP	10.1155/2014/541370
9	2015	Yusuf Pandir; Ayse Yildirim	Space-time fractional potential KP	Extended tanh method; modified Riemann-Liouville derivative	Exact fractional KP travelling-wave solutions	No extensive numerical validation	10.1063/1.4912605
10	2016	E. M. E. Zayed; K. A. E. Alurrfi	Extended auxiliary equation method	Auxiliary equation and Jacobi elliptic functions	New elliptic and solitary solutions	NLS rather than KP	10.1016/j.amc.2016.04.014
11	2017	E. M. E. Zayed; Abdul-Ghani Al-Nowehy	Extended auxiliary equation method	Jacobi elliptic-function expansion	New periodic and solitary solutions	NLS rather than KP	10.1080/17455030.2016.1259690
12	2017	Syed Tauseef Mohyud-Din; Touqeer Nawaz; Ehtsham Azhar; M. Ali Akbar	Fractional potential KP and Calogero-Degasperis equations	Fractional sub-equation method	Hyperbolic, trigonometric and rational solutions	Modified Riemann-Liouville formulation	10.1016/j.jtusc.2014.11.010
13	2022	Yanfeng Guo; Zhengde Dai; Chunxiao Guo	Lump and interaction solutions of KPI	Hirota bilinear method and interaction analysis	Lump, lump-soliton and breather solutions	Classical KPI	10.1007/s11464-021-0973-y
14	2024	Abaker Hassaballa et al.	Space-time fractional KP	Conformable derivative; (G'/G)-expansion	Exact fractional travelling-wave solutions	Limited numerical/derivative comparison	10.3389/fams.2024.1379937
15	2024	Handan Borluk; Gabriele Bruell; Dag Nilsson	Lump solutions of fractional KP	Variational methods; Petviashvili iteration	Existence, smoothness and decay of fKPI lumps	No closed-form exact solutions	10.1007/s13540-023-00236-2
16	2024	Uttam Kumar Mandal et al.	Extended (3+1)-dimensional B-type KP	Bell polynomials, Hirota method, Bäcklund transformation, Lax pair, Lie symmetry	Integrability and rational, breather, soliton solutions	Classical, not fractional	10.1016/j.chaos.2024.114689

17	2026	Sarasvati Yadav; Vishal Chauhan	Extended Camassa-Holm-KP equation	Lie symmetry, invariant subspaces, conservation laws	Symmetry reductions and soliton/periodic solutions	Classical equation	10.1007/s11071-025-11959-7
18	2006	Gino Biondini; S. Chakravarty	KPII soliton solutions	Wronskian/ τ -function formulation	Line and multisoliton solutions	Classical KPII	10.1063/1.2181907
19	2009	Sarbarish Chakravarty; Yuji Kodama	KP solitons and shallow-water waves	Grassmannian/Schubert-cell classification	Systematic classification of KP solitons	Classical KP	10.1111/j.1467-9590.2009.00448.x
20	2010	E. J. Parkes	tanh-coth expansion method	Analytical examination of tanh-coth method	Clarifies relationships among expansion methods	General nonlinear equations	10.1016/j.amc.2009.11.037
21	2011	Ahmet Bekir; Adem C. Cevikel	Nonlinear coupled/breaking soliton equations	tanh-coth and Riccati method	Exact travelling-wave and soliton solutions	Classical equations	10.1016/j.jksus.2010.06.020
22	2022	Handan Borluk; Gabriele Bruell; Dag Nilsson	Fractional KP travelling waves and instability	Bifurcation, Petviashvili iteration, exponential time differencing	Existence and transverse stability/instability	Focus on stability rather than closed forms	10.1111/sapm.12494
23	2023	Gui-Qiong Xu; Abdul-Majid Wazwaz	$(n + 1)$ -dimensional generalized KP	Bell polynomials, Painlevé analysis, Lax pair, Bäcklund transformation	N-solitons, lumps, breathers and localized solutions	Classical generalized KP	10.1007/s11071-023-08343-8
24	2026	Saud Owyed	New wave patterns for $(3+1)$ -dimensional KP	Generalized Arnous and $(G'/(bG' + G + a))$ -expansion methods	Rational, trigonometric, hyperbolic, bright/dark and singular waves	Classical KP; no fractional effects	10.3934/math.2026086
25	2026	Sujoy Devnath; Khaled A. Gepreel; Hassan M. Aljohani; M. Ali Akbar; Sachin Kumar	Time-fractional $(3+1)$ -dimensional KP-I	$(G'/G, 1/G)$ -expansion; improved F-expansion; beta fractional derivative; dynamical analysis	Hyperbolic, trigonometric and algebraic soliton solutions	No systematic comparison of fractional derivatives	10.1007/s13137-026-00286-7
26	2024	Tianle Yin; Jing Pang	Variable-coefficient $(2+1)$ -dimensional KP for Rossby waves	Multiscale analysis, perturbation, auxiliary equation, modified Hirota method	Rossby solitons, rogue waves, lumps and interactions	Classical variable-coefficient KP	10.1007/s11071-023-09177-0

27	2015	Chuanjian Wang; Zhengde Dai; Changfu Liu	Breather-like and rational solutions of KP-based system	Hirota bilinear method; long-wave limit	Breather-like, rational, lump and solitary-wave structures	Classical KP-based system	10.1155/2015/861069
28	2010	C. M. Khaliq ; A. R. Adem	Generalized (3+1)-dimensional KP exact solutions	Lie symmetry analysis and symmetry reductions	Exact solutions using Lie symmetries	Classical KP; no fractional effects	10.1016/j.amc.2010.03.135
29	2011	Long Wei	Kadomtsev-Petviashvili-Burgers equation	Lie symmetry analysis, symmetry reduction, Exp-function method	Exact solitary-wave and periodic-wave solutions	Classical KP-Burgers equation; no fractional derivative or memory effects	10.1134/S0965542511080148
30	2013	Abdullahi Rashid Adem; Chaudry Masood Khaliq	Coupled Kadomtsev-Petviashvili system	Lie symmetry method; (G'/G)-expansion method; multiplier method	Obtains travelling-wave exact solutions and derives conservation laws for a coupled KP system	Classical coupled KP system; no fractional derivative or fractional-memory effects	10.1016/j.compfluid.2013.04.005

Mathematical Preliminaries

This section presents the fractional operators and notation required for the formulation and analysis of the Caputo space-time fractional Kadomtsev-Petviashvili equation [9,12,14,15,22-25].

Let $\Omega \subset \mathbb{R}^2$ be a bounded spatial domain and let $u : \Omega \times [0, T] \rightarrow \mathbb{R}$

Be sufficiently smooth such that all fractional derivatives appearing in the model exist and belong to the required function spaces.

Gamma Function

The Gamma function is defined for $\alpha > 0$ by

$$\Gamma(\alpha) = \int_0^{\infty} t^{\alpha-1} e^{-t} dt \quad (1)$$

And satisfies the recurrence relation

$$\Gamma(\alpha + 1) = \alpha \Gamma(\alpha). \quad (2)$$

In particular,

$$\Gamma(n + 1) = n! \quad (3)$$

For every non-negative integer

Riemann-Liouville Fractional Integral

For $\alpha > 0$, the Riemann-Liouville fractional integral of a function $f \in L^1([0, T])$ is defined by

$$I_{\alpha}^{\eta} f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (\alpha - \tau)^{\alpha-1} f(\tau) d\tau \quad (4)$$

Caputo Fractional Derivative

Let $n - 1 < \alpha < n$, where $n \in \mathbb{N}$. The Caputo fractional derivative of order α is defined as

$${}^c D_t^\alpha f(t) = \frac{1}{\Gamma(n - \alpha)} \int_0^t (t - \tau)^{n - \alpha - 1} f^{(n)}(\tau) d\tau \quad (5)$$

For the present study, only the case $0 < \alpha \leq 1$ is required, and the definition reduces to

$${}^c D_t^\alpha f(t) = \frac{1}{\Gamma(1 - \alpha)} \int_0^t (t - \tau)^{-\alpha} f'(\tau) d\tau \quad (6)$$

The Caputo derivative satisfies ${}^c D_t^\alpha C = 0$ For any constant C , and for $m \geq 0$,

$${}^c D_t^\alpha t^m = \frac{\Gamma(m + 1)}{\Gamma(m + 1 - \alpha)} \cdot t^{m - \alpha}. \quad (7)$$

Moreover,

$$\lim_{\alpha \rightarrow 1} {}^c D_t^\alpha f(t) = \frac{df(t)}{dt}, \quad (8)$$

Showing that the classical derivative is recovered as the fractional order approaches unity [9], [12], [14], [15], [22], [25].

Fractional Spatial and Temporal Operators

For $u = u(x, y, t)$, the temporal Caputo operator of order $0 < \alpha \leq 1$ is defined by

$${}^c D_t^\alpha u(x, y, t) = \frac{1}{\Gamma(1 - \alpha)} \int_0^t (t - \tau)^{-\alpha} \left(\frac{\partial u(x, y, \tau)}{\partial \tau} \right) d\tau \quad (9)$$

Similarly, the spatial Caputo operators are

$${}^c D_x^\beta u(x, y, t) = \frac{1}{\Gamma(1 - \beta)} \int_0^x (x - \xi)^{-\beta} \left(\frac{\partial u(\xi, y, t)}{\partial \xi} \right) d\xi \quad (10)$$

And

$${}^c D_y^\gamma u(x, y, t) = \frac{1}{\Gamma(1 - \gamma)} \int_0^y (y - \zeta)^{-\gamma} \left(\frac{\partial u(x, \zeta, t)}{\partial \zeta} \right) d\zeta \quad (11)$$

Where $0 < \beta \leq 1$ and $0 < \gamma \leq 1$.

The higher-order operators appearing in the fractional KP equation are interpreted as sequential Caputo derivatives:

$${}^c D_x^{2\beta} u = {}^c D_x^\beta ({}^c D_x^\beta u), \quad (12)$$

$${}^c D_x^{4\beta} u = {}^c D_x^\beta ({}^c D_x^\beta ({}^c D_x^\beta ({}^c D_x^\beta u))). \quad (13)$$

The mixed space-time operator is defined by ${}^c D_t^\alpha$

$${}^c D_x^\beta ({}^c D_t^\alpha u), \quad (14)$$

This describes the coupled nonlocal evolution of the wave field in both space and time.

Throughout this work, the fractional orders satisfy $0 < \alpha, \beta, \gamma \leq 1$,

Where α characterises temporal memory effects, β governs the longitudinal spatial nonlocality, and γ controls the transverse spatial dispersion.

When $\alpha \rightarrow 1, \beta \rightarrow 1, \gamma \rightarrow 1$, the fractional operators reduce to the classical derivatives:

$${}^c D_t^\alpha \rightarrow \frac{\partial}{\partial t}, \quad (15)$$

$${}^c D_x^\beta \rightarrow \frac{\partial}{\partial x}, \quad (16)$$

$${}^c D_y^\gamma \rightarrow \frac{\partial}{\partial y}. \quad (17)$$

These preliminaries provide the mathematical foundation for the formulation and analytical treatment of the Caputo

space-time fractional Kadomtsev-Petviashvili equation considered in this study

Governing Equation

We consider the classical Kadomtsev-Petviashvili (KP) equation

$$u_{tx} - uu_{xx} - (u_x)^2 - u_{xxx} = u_{yy} \quad (18)$$

Where $u(x, y, t)$ denotes the wave field and the subscripts indicate partial differentiation with respect to the corresponding independent variables. The classical KP model describes multidimensional nonlinear dispersive-wave propagation by incorporating weak transverse effects into the underlying one-dimensional KdV dynamics.

To incorporate temporal memory and fractional spatial effects, the integer-order derivatives are generalized through Caputo fractional operators. The resulting Caputo space-time fractional KP equation is

$${}^c D_x^\beta ({}^c D_t^\alpha u) - u({}^c D_x^{2\beta} u) - ({}^c D_x^\beta u)^2 - {}^c D_x^{4\beta} u = {}^c D_y^\gamma u \quad (19)$$

Where $0 < \alpha, \beta, \gamma \leq 1$.

Where α, β , and γ denote the fractional orders associated with the temporal variable t and the spatial variables x and y , respectively, with the corresponding admissible ranges specified in the model formulation [9], [12], [14], [25].

Eq. (2) extends the classical KP framework by introducing nonlocal fractional operators capable of representing memory and hereditary effects in the evolution dynamics. In this formulation, the fractional orders provide additional control over the temporal and spatial scaling of the travelling-wave structures while retaining the classical model as the appropriate integer-order limit [9], [14], [15], [22], [25].

The subsequent analysis applies the Fractional Complex Transformation to reduce Eq. (2) to a nonlinear ordinary differential equation. The reduced equation is then treated using the Riccati Equation Method and the Extended Auxiliary Equation Method to derive and compare exact travelling-wave solutions belonging to hyperbolic, trigonometric/periodic, and rational regimes [6]-[12], [14], [20], [21], [25].

Formulation and Fractional Travelling-Wave Reduction

The classical Kadomtsev-Petviashvili (KP) equation is given by

$$(u_t + uu_x + u_{xxx})_x + u_{yy} = 0. \quad (20)$$

Expanding the derivative this yields to the governing equation i.e eq(1)

$$u_{tx} + uu_{xx} + (u_x)^2 + u_{xxx} + u_{yy} = 0.$$

To incorporate temporal memory and spatial non-locality effects, the integer-order derivatives are replaced by Caputo fractional operators. The resulting Caputo space-time fractional KP equation is formulated as, Eq. (2)

$${}^c D_x^\beta ({}^c D_t^\alpha u) - u({}^c D_x^{2\beta} u) - ({}^c D_x^\beta u)^2 - {}^c D_x^{4\beta} u = {}^c D_y^{2\gamma} u,$$

Where $u = u(x, y, t)$, $0 < \alpha, \beta, \gamma \leq 1$.

Throughout this work, it is assumed that $u \in C^4(\Omega \times [0, T])$

And that all required sequential Caputo derivatives exist [9], [12], [14], [15], [22], [25].

Fractional Travelling-Wave Transformation

To reduce the fractional partial differential equation to an ordinary differential equation, we introduce the fractional travelling-wave transformation

$$u(x, y, t) = U(\xi), \quad (21)$$

Where the wave variable is defined by

$$\xi = \frac{k}{\Gamma(1+\beta)} \cdot x^\beta + \frac{l}{\Gamma(1+\gamma)} \cdot y^\gamma - \frac{c}{\Gamma(1+\alpha)} \cdot t^\alpha, \quad (22)$$

With $a, b, c \neq 0$ representing wave parameters.

Using the Caputo derivative identity

$${}^c D_x^\beta \left(\frac{x^\beta}{\Gamma(1+\beta)} \right) = 1, \quad (23)$$

The transformed derivatives become

$${}^c D_t^\alpha u = -cU', \quad (24)$$

$${}^c D_x^\beta u = kU', \quad (25)$$

$${}^c D_y^\gamma u = lU', \quad (26)$$

Where the prime denotes differentiation with respect to ξ .

The sequential higher-order derivatives are therefore

$${}^c D_x^{2\beta} u = k^2 U'', \quad (27)$$

$${}^c D_x^{4\beta} u = k^4 U^{(4)}, \quad (28)$$

And the mixed space-time derivative becomes

$${}^c D_x^\beta ({}^c D_t^\alpha u) = -kcU''. \quad (29)$$

Similarly,

$${}^c D_y^{2\gamma} u = l^2 U''. \quad (30)$$

Substituting these expressions into the fractional KP equation gives

$$-kcU'' - k^2 U U'' - k^2 (U')^2 - k^4 U^{(4)} = k^2 U''. \quad (31)$$

Rearranging, we obtain

$$k^4 U^{(4)} + k^2 U U'' + k^2 (U')^2 + (kc + l^2) U'' = 0. \quad (32)$$

Observing that

$$\frac{d^2}{d\xi^2} \left(\frac{U^2}{2} \right) = U U'' + (U')^2, \quad (33)$$

The equation can be written as

$$k^4 U^{(4)} + \frac{k^2 d^2}{d\xi^2 \left(\frac{U^2}{2}\right)} + (kc + l^2)U'' = 0. \quad (34)$$

Integrating twice with respect to ξ yields

$$k^4 U'' + \frac{k^2}{2} \cdot U^2 + (kc + l^2)U = C_1 \xi + C_2, \quad (35)$$

Where C_1 and C_2 are integration constants.

For localized travelling-wave solutions satisfying $U(\xi) \rightarrow 0, U'(\xi) \rightarrow 0, U''(\xi) \rightarrow 0, |\xi| \rightarrow \infty$, the integration constants vanish: $C_1 = C_2 = 0$.

Consequently, the reduced nonlinear ordinary differential equation becomes

$$k^4 U'' + \frac{k^2}{2} \cdot U^2 + (kc + l^2)U = 0. \quad (36)$$

This second-order nonlinear ordinary differential equation constitutes the fundamental reduced model to which the Riccati equation method and the extended auxiliary equation method will be applied in order to construct exact analytical travelling-wave solutions [6], [8], [10], [11].

Analytical Solution Methodology

Consider the reduced nonlinear ordinary differential equation

$$k^4 U'' + \frac{k^2}{2} \cdot U^2 + (kc + l^2)U = 0.$$

Homogeneous Balance

To determine the appropriate structure of the analytical ansatz, we balance the highest derivative term U'' with the nonlinear term U^2 . Assuming

$$U(\xi) \sim \phi(\xi)^N \quad (37)$$

And using the Riccati relation

$$\phi' = \phi^2 + \lambda, \quad (38)$$

It follows that

$$U'' \sim \phi^{N+2}, \quad (39)$$

$$U^2 \sim \phi^{2N}. \quad (40)$$

Balancing the dominant powers gives

$$N + 2 = 2N, \quad (41)$$

Which yield $N = 2$.

Therefore, the appropriate finite expansion is

$$U(\xi) = A_0 + A_1 \phi(\xi) + A_2 \phi^2(\xi), \quad (42)$$

Where A_0, A_1 , and A_2 are constants to be determined.

Riccati Equation Method

The auxiliary function $\phi(\xi)$ is assumed to satisfy the Riccati equation

$$\phi'(\xi) = \phi^2(\xi) + \lambda, \quad (43)$$

Where λ is a constant parameter.

Differentiating Eq. (43) gives

$$\phi''(\xi) = 2\phi(\xi)(\phi'(\xi) + \lambda). \quad (44)$$

Substituting the expansion Eq. (42) together with Eqs. (43)-(44) into Eq. (36) produces a polynomial in powers of ϕ . Equating the coefficients of like powers of ϕ yields a nonlinear algebraic system for the unknown parameters A_0, A_1, A_2 , and λ .

The Riccati equation admits different classes of solutions according to the sign of λ [6], [7], [20], [21].

(i) $\lambda > 0$

$$\phi(\xi) = -\sqrt{\lambda} \cot(\sqrt{\lambda}\xi). \quad (45)$$

(ii) $\lambda < 0$

$$\phi(\xi) = -\sqrt{-\lambda} \tanh(\sqrt{-\lambda}\xi), \quad (46)$$

$$\phi(\xi) = -\sqrt{-\lambda} \coth(\sqrt{-\lambda}\xi). \quad (47)$$

(iii) $\lambda = 0$

$$\phi(\xi) = -\frac{1}{\xi}. \quad (48)$$

Substituting these expressions into Eq. (42) generates periodic, solitary, and rational travelling-wave solutions of the reduced equation.

Extended Auxiliary Equation Method

To obtain a broader Regimes of exact solutions, we further assume the representation

$$U(\xi) = A_0 + A_1\psi(\xi) + A_2\psi^2(\xi), \quad (49)$$

Where $\psi(\xi)$ satisfies the quartic auxiliary equation

$$(\psi')^2 = P_0 + P_1\psi + P_2\psi^2 + P_3\psi^3 + P_4\psi^4, \quad (50)$$

With P_0, P_1, P_2, P_3 , and P_4 being constants.

Differentiating Eq. (52) with respect to ξ yields

$$\psi'' = \frac{1}{2}(P_1 + 2P_2\psi + 3P_3\psi^2 + 4P_4\psi^3). \quad (51)$$

Substituting Eqs. (49)-(51) into Eq. (36) and collecting coefficients of equal powers of ψ produces a nonlinear algebraic system whose solution determines the admissible parameter sets $(A_0, A_1, A_2, P_0, P_1, P_2, P_3, P_4)$.

Depending on the resulting parameter configuration, Eq. (50) generates several classes of exact solutions, including hyperbolic, trigonometric, and rational wave structures [8], [10], [11].

The corresponding solutions of the original Caputo space-time fractional KP equation are obtained through the inverse travelling-wave transformation

$$\xi = \frac{k}{\Gamma(1+\beta)} \cdot x^\beta + \frac{l}{\Gamma(1+\gamma)} \cdot y^\gamma - \frac{c}{\Gamma(1+\alpha)} \cdot t^\alpha. \quad (52)$$

These analytical techniques provide complementary mechanisms for constructing closed-form travelling-wave solutions while preserving the influence of the fractional parameters through the fractional wave coordinate [8]-[12], [14].

Exact Travelling-Wave Solutions

The reduced nonlinear ordinary differential equation obtained from the fractional complex transformation is given by

$$k^4 U'' + \left(\frac{k^2}{2}\right) U^2 + (kc + l^2)U = 0. \quad (53)$$

The exact travelling-wave solutions of the Caputo space-time fractional KP equation are constructed using two analytical frameworks: the Riccati equation method and the extended auxiliary equation method. Although both approaches generate the same fundamental classes of nonlinear wave structures, namely hyperbolic (solitary), trigonometric (periodic), and rational solutions, the corresponding functional forms differ due to the distinct auxiliary equations employed [6]-[8], [10], [11], [20], [21].

The fractional travelling-wave coordinate is defined as

$$\xi = \left(\frac{k}{\Gamma(1+\beta)}\right) x^\beta + \left(\frac{l}{\Gamma(1+\gamma)}\right) y^\gamma - \left(\frac{c}{\Gamma(1+\alpha)}\right) t^\alpha. \quad (54)$$

Exact Solutions Obtained by the Riccati Equation Method

The Riccati method assumes the solution representation

$$U(\xi) = A_0 + A_1 \phi(\xi) + A_2 \phi^2(\xi), \quad (55)$$

Where the auxiliary function $\phi(\xi)$ satisfies the Riccati equation

$$\phi'(\xi) = \phi^2(\xi) + \lambda, \quad (56)$$

With λ being an arbitrary constant parameter.

Substitution of Eq. (55) into the reduced Eq. (53), together with Eq. (56), generates an algebraic system for the parameters A_0, A_1, A_2 , and λ . The resulting admissible parameter sets produce three classes of travelling-wave solutions [6], [7], [20], [21].

Solitary-Wave Solution (Riccati Method)

For the case $\lambda < 0$, The Riccati equation admits the hyperbolic solution

$$\phi(\xi) = -\sqrt{-\lambda} \tanh(\sqrt{-\lambda} \xi). \quad (57)$$

Substituting Eq. (59) into Eq. (57) gives the solitary-wave solution

$$U_{s1}(\xi) = A_0 - A_1\sqrt{-\lambda}\tanh(\sqrt{-\lambda}\xi) - A_2\lambda\tanh^2(\sqrt{-\lambda}\xi). \quad (58)$$

and

$$U_{s2}(\xi) = A_0 - A_1\sqrt{-\lambda}\coth(\sqrt{-\lambda}\xi) - A_2\lambda\coth^2(\sqrt{-\lambda}\xi), \quad (59)$$

Therefore, the corresponding solitary-wave solutions of the fractional KP equation is

$$u_{si}(x, y, t) = U_{si}(\xi). \quad (60)$$

This solution represents a localized nonlinear travelling-wave structure generated by the balance between fractional dispersion and nonlinear interaction.

Periodic-Wave Solution (Riccati Method)

For the condition $\lambda > 0$, The auxiliary function takes the trigonometric form

$$\phi(\xi) = \sqrt{\lambda}\tan(\sqrt{\lambda}\xi). \quad (61)$$

Consequently, the periodic travelling-wave solution becomes

$$U_{p1}(\xi) = A_0 + A_1\sqrt{\lambda}\tan(\sqrt{\lambda}\xi) + A_2\lambda\tan^2(\sqrt{\lambda}\xi). \quad (62)$$

and

$$U_{p3}(\xi) = A_0 - A_1\sqrt{\lambda}\cot(\sqrt{\lambda}\xi) + A_2\lambda\cot^2(\sqrt{\lambda}\xi) \quad (63)$$

Hence, the corresponding periodic-wave solutions of the fractional KP equation is

$$u_{pi}(x, y, t) = U_{pi}(\xi). \quad (64)$$

This Regimes describes oscillatory travelling-wave structures associated with the periodic branch of the Riccati equation [6], [20], [21].

Rational Solution (Riccati Method)

For the limiting case $\lambda = 0$, The Riccati equation reduces to

$$\phi(\xi) = -\frac{1}{\xi}. \quad (65)$$

The corresponding rational solution is

$$U_{r1}(\xi) = A_0 - \frac{A_1}{\xi} + \frac{A_2}{\xi^2}. \quad (66)$$

Therefore,

$$u_{ri}(x, y, t) = U_{ri}(\xi). \quad (67)$$

This solution represents an algebraically decaying travelling-wave structure.

Graphical representation of Riccati obtained solutions

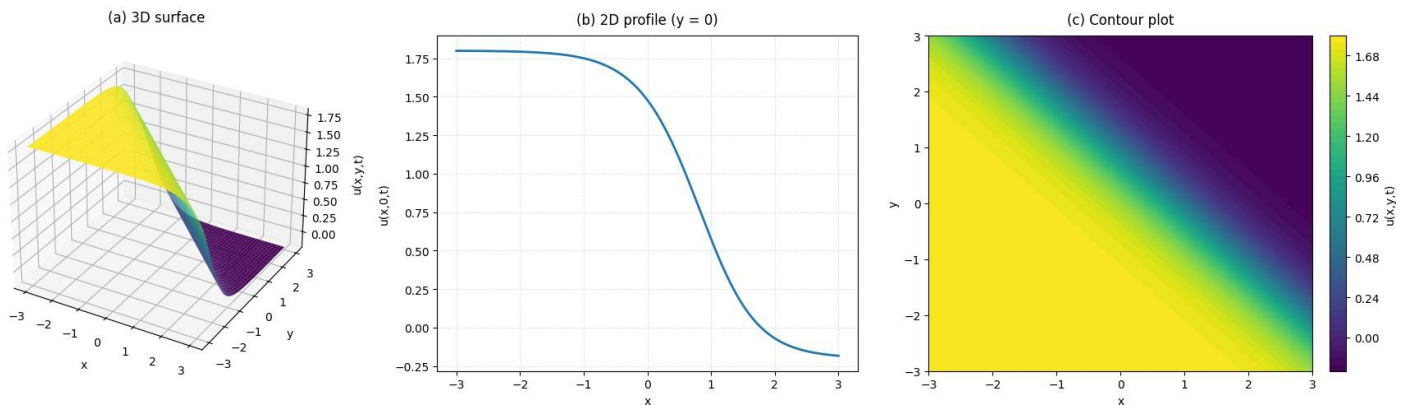
Riccati tanh Hyperbolic-Wave Solution $U_{(s_1)}$ | $\alpha = 0.8$, $\beta = 1.0$, $\gamma = 1.0$, $t = 1.0$

Figure 1: Riccati *Tanh* Hyperbolic-Wave Solution Regimes

Figure 1: The 3-D surface, 2-D profile and Contour plots of the travelling-wave solution U_{S_1} of the Caputo time- fractional Kadomtsev-Petviashvili equation are shown to display travelling wave structure that is smooth and bounded. The finite asymptotic behaviour of the *tanh* profile indicates a regular hyperbolic structure of the profile, which is related to the balance between nonlinear and dispersive effects, and the fractional order α changes the propagation and temporal evolution of the profile without altering the underlying *tanh* functional form.

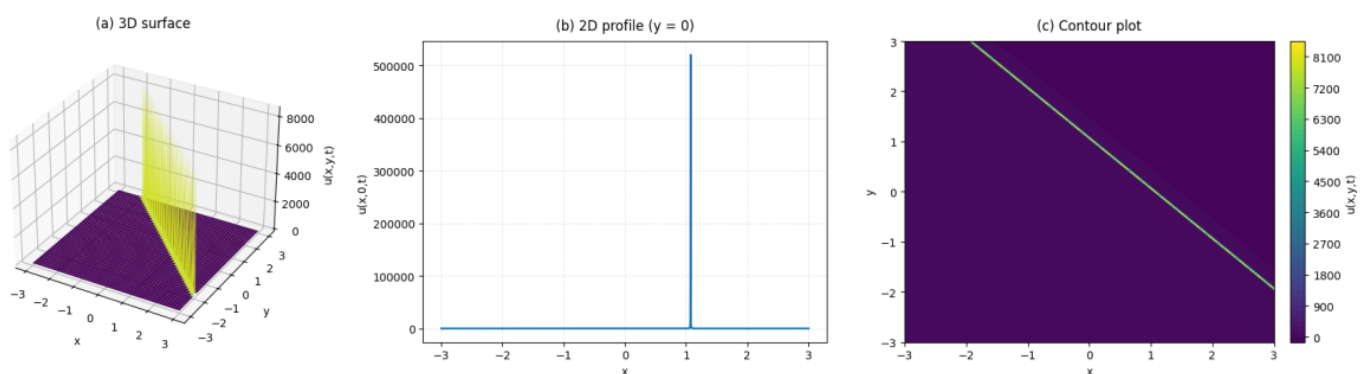
Riccati Coth Hyperbolic-Wave Solution $U_{(s_3)}$ | $\alpha = 0.8$, $\beta = 1.0$, $\gamma = 1.0$, $t = 1.0$


Figure 2: The 3-D surface plot, 2-D profile plot, and Contour plot of the Riccati *coth* hyperbolic-wave solution U_{S_3} of the Caputo time-fractional Kadomtsev-Petviashvili equation illustrate a singular travelling-wave solution whose shape is that of a hyperbolic *coth*. The steep amplitude variation near the coth pole is associated with the singular character of this hyperbolic branch and the fractional order α modifies its temporal evolution and propagation properties but not its *coth* functional form.

Figure 3- Riccati Tan Periodic-Wave Solution Regimes

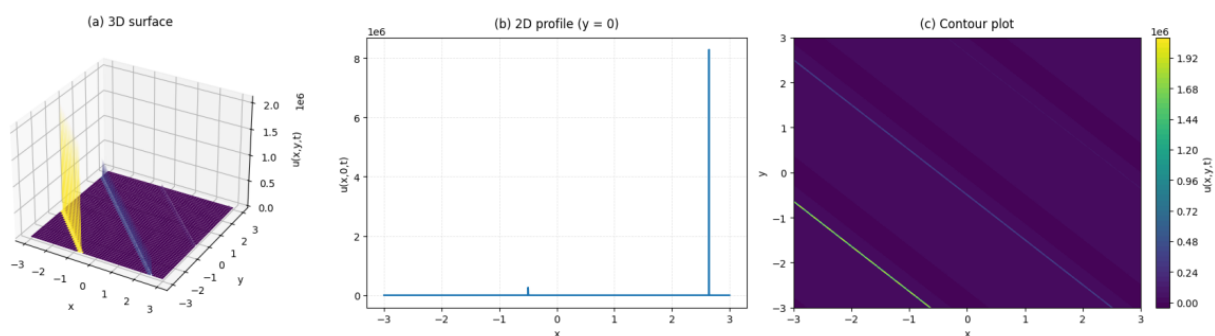
Riccati Tan Periodic-Wave Solution $U_{(p_1)}$ | $\alpha = 0.8$, $\beta = 1.0$, $\gamma = 1.0$, $t = 1.0$


Figure 3: Shows the periodic travelling wave solution of U_{P1} in the Caputo time-fractional Kadomtsev-Petviashvili equation in the 3D surface plot, 2D profile plot and Contour plot respectively. The regular occurrence of wave patterns and regularly spaced singular points are due to the fact that the tan branch is one of the trigonometric functions. The fractional order α , on the other hand, influences the evolution of the system in space and time but leaves the periodic functional form unchanged.

Riccati Cot Periodic-Wave Solution $U_{(P3)}$ | $\alpha = 0.8, \beta = 1.0, \gamma = 1.0, t = 1.0$

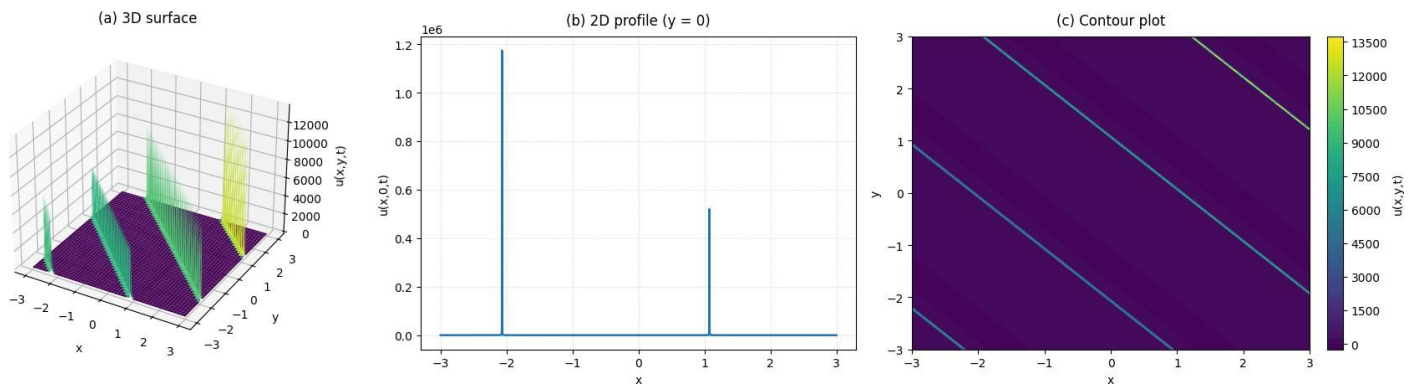


Figure 4: The 3D surface, 2D profile, and Contour views of the Riccati periodic wave solution U_{P3} of the Caputo time-fractional Kadomtsev-Petviashvili equation show a periodic travelling-wave solution with regularly distributed singular points in the solution. The periodic functional form of the wave pattern recurs at the zeros of $\sin(\sqrt{\lambda}\xi)$ and is modulated by the fractional order α which affects the temporal evolution and phase position but not the periodic functional form. Figure 5- Riccati Rational-Wave Solution Regimes

Riccati Rational-Wave Solution $U_{(R1)}$ | $\alpha = 0.8, \beta = 1.0, \gamma = 1.0, t = 1.0$

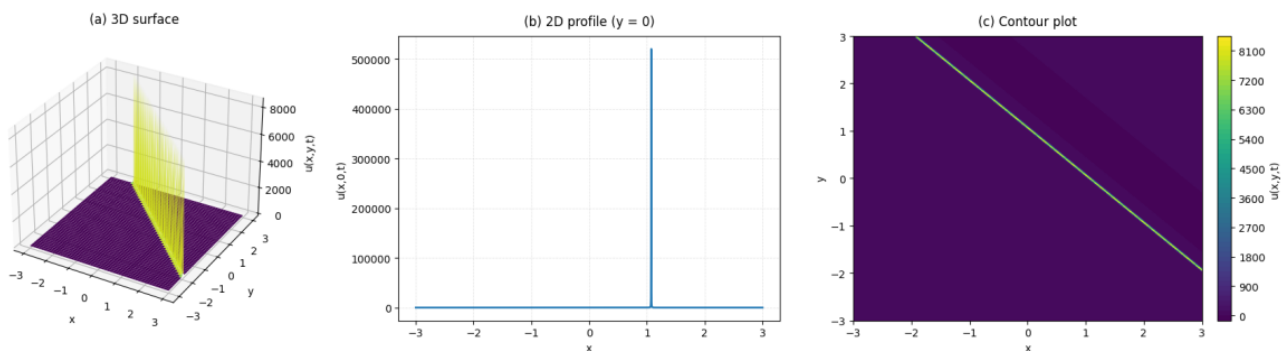


Figure 5: Shows the 3D surface plot, the 2D profile plot and the Contour plot of the non-periodic inverse-power travelling-wave solution U_{R1} to the Caputo time-fractional Kadomtsev-Petviashvili equation. The solution is algebraically varying away from the singular travelling coordinate $\xi = 0$ and converges towards the background level $A0$ with increasing $|\xi|$, with the fractional order α affecting its temporal evolution and propagation properties without changing the underlying rational functional form of the solution.

Exact Solutions Obtained by the Extended Auxiliary Equation Method

To obtain additional solution families, the extended auxiliary equation method is applied. The solution is assumed in the form

$$U(\xi) = A_0 + A_1\psi(\xi) + A_2\psi^2(\xi), \quad (68)$$

Where the auxiliary function satisfies the quartic equation

$$(\psi')^2 = P_0 + P_1\psi + P_2\psi^2 + P_3\psi^3 + P_4\psi^4. \quad (69)$$

The substitution of Eqs. (68)-(69) into the reduced Eq. (53) yields an algebraic system determining the parameters

$A_0, A_1, A_2, P_0, P_1, P_2, P_3$, And P_4 .

Depending on the admissible parameter configurations, three solution families are obtained.

Solitary-Wave Solution (Extended Auxiliary Method)

For parameter conditions generating hyperbolic auxiliary functions,

$$\psi(\xi) = \tanh(k\xi), \quad (70)$$

and

$$\psi(\xi) = \coth(k\xi), \quad (71)$$

The travelling-wave solution takes the form

$$U_{s2}(\xi) = A_0 + A_1\tanh(k\xi) + A_2\tanh^2(k\xi). \quad (72)$$

$$U_{s3}(\xi) = A_0 + A_1\coth(k\xi) + A_2\coth^2(k\xi) \quad (73)$$

The corresponding solitary-wave solutions of the fractional KP equation is

$$u_{s2}(x, y, t) = U_{s2}(\xi). \quad (74)$$

Periodic-Wave Solution (Extended Auxiliary Method)

For parameter configurations producing trigonometric auxiliary functions,

$$\psi(\xi) = \tan(k\xi), \quad (75)$$

and

$$\psi(\xi) = \cot(k\xi), \quad (76)$$

The solution becomes

$$U_{p2}(\xi) = A_0 + A_1\tan(k\xi) + A_2\tan^2(k\xi). \quad (77)$$

$$U_{p3}(\xi) = A_0 + A_1\cot(k\xi) + A_2\cot^2(k\xi) \quad (78)$$

Hence, the corresponding periodic-wave solutions of the fractional KP equation is

$$u_{pi}(x, y, t) = U_{pi}(\xi). \quad (79)$$

Rational Solution (Extended Auxiliary Method)

For degenerate parameter conditions, the auxiliary function reduces to a rational form

$$\psi(\xi) = \frac{1}{\xi}. \quad (80)$$

Therefore, the rational travelling-wave solution is

$$U_{r2}(\xi) = A_0 + \frac{A_1}{\xi} + \frac{A_2}{\xi^2}. \quad (81)$$

The corresponding fractional KP solution is

$$ur_i(x, y, t) = Ur_i(\xi). \quad (82)$$

Graphical representation of Extended Auxiliary Equation obtained solutions.

Figure 6- Extended Auxiliary *Tanh* Hyperbolic-Wave Solution Regimes

Extended Auxiliary Tanh Hyperbolic-Wave Solution U_{S2} | $\alpha = 0.8, \beta = 1.0, \gamma = 1.0, t = 1.0$

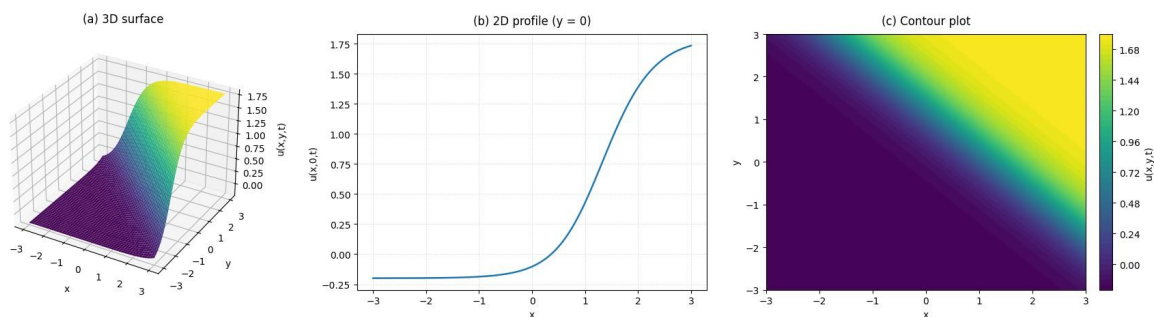


Figure 6: The 3D surface, 2D profile and Contour plots of the traveling-wave structure solution U_{S2} of the Caputo time-fractional Kadomtsev-Petviashvili equation shows a smooth and bounded travelling-wave structure. The finite asymptotic behaviour of the *tanh* profile is in accordance with a regular hyperbolic structure, which is related to the balance between nonlinear and dispersive effects, and the fractional order α also affects the temporal evolution and propagation of the profile, but does not change its functional form.

Figure 7- Extended Auxiliary *Coth* Hyperbolic-Wave Solution Regimes

Extended Auxiliary Coth Hyperbolic-Wave Solution U_{S3} | $\alpha = 0.8, \beta = 1.0, \gamma = 1.0, t = 1.0$

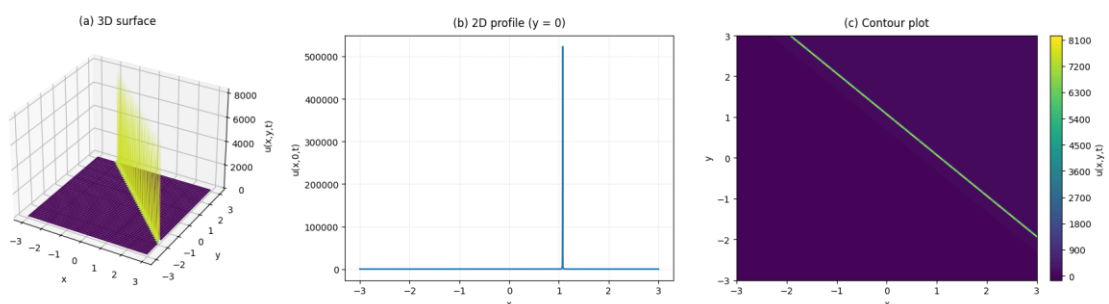


Figure 7: The Extended Auxiliary *coth* hyperbolic-wave solution U_{S3} for the special case of the Caputo time-fractional Kadomtsev-Petviashvili equation is shown in the three representations: 3D surface, 2D profile and Contour. The amplitude variation is steep in the vicinity of the *coth* pole and the fractional order α is introduced to modulate the temporal evolution and propagation characteristics of the *coth* underlying functional form without changing it.

Figure 8- Extended Auxiliary *Tan* Periodic-Wave Solution Regimes

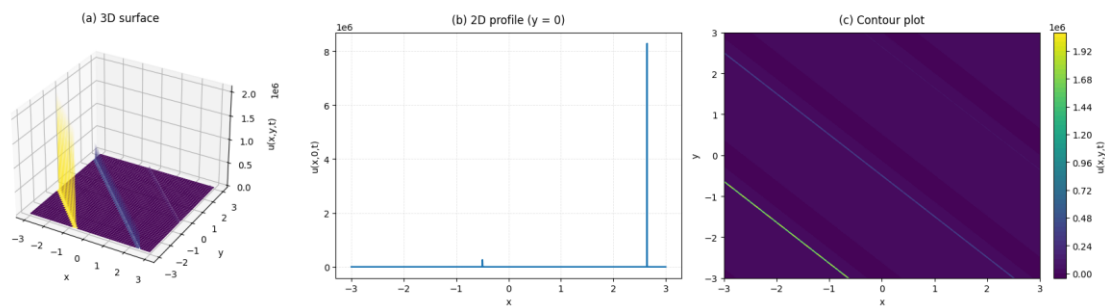
Extended Auxiliary Tan Periodic-Wave Solution U_{p2} | $\alpha = 0.8, \beta = 1.0, \gamma = 1.0, t = 1.0$


Figure 8: In the 3D surface, 2D profile and Contour representations of periodic travelling-wave solution U_{p2} of the Extended Auxiliary, regularly distributed singular points are presented in the Caputo time-fractional Kadomtsev- Petviashvili equation. The periodic functional form of the wave is singularities at the zeros of $\cos(k\xi)$, while the fractional order α only affects the temporal evolution and the phase position of the wave.

Figure 9- Extended Auxiliary Cot Periodic-Wave Solution Regimes

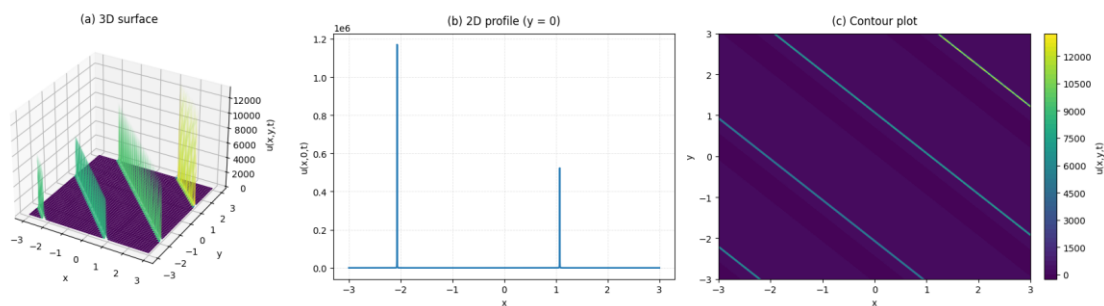
Extended Auxiliary Cot Periodic-Wave Solution U_{p3} | $\alpha = 0.8, \beta = 1.0, \gamma = 1.0, t = 1.0$


Figure 9: The 3D surface, 2D profile and Contour plots of the Extended Auxiliary periodic-wave solution U_{p3} of the Caputo time-fractional Kadomtsev-Petviashvili equation show a periodic travelling-wave structure with regularly distributed singular points. The periodic functional form is repeated in the form of recurring wave patterns, the zeros of $\sin(k\xi)$ are the singularities, and the fractional order α modulates the temporal evolution and phase position of the periodic functional form without altering its form.

Figure 10- Extended Auxiliary Rational-Wave Solution Regimes

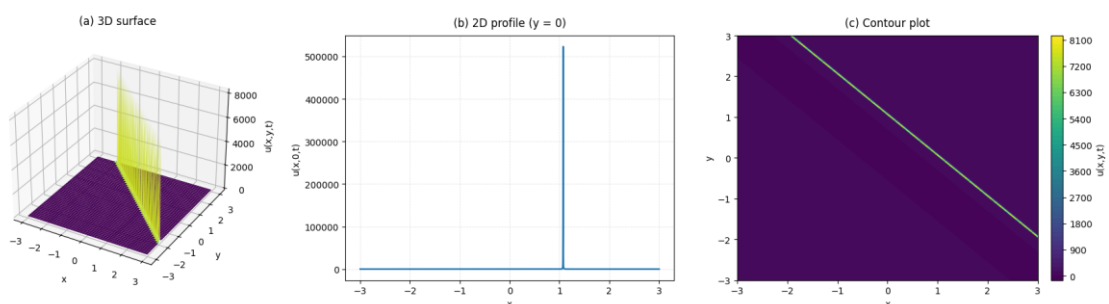
Extended Auxiliary Rational-Wave Solution U_{r2} | $\alpha = 0.8, \beta = 1.0, \gamma = 1.0, t = 1.0$


Figure 10: The 3D surface and 2D profile of the solution U_{r2} for the Extended Auxiliary rational-wave solution to the Caputo time-fractional KP equation shows the structure of the inverse-power travelling wave solution in the Caputo time- fractional KP equation, which is non-periodic. The algebraic variation from $\xi = 0$, the singular travelling coordinate, is a clue to the rational solution; the fractional order α is a modulator of the temporal evolution and propagation properties of the solution without altering the rational functional form of its solution.

COMPARATIVE RESULTS AND DISCUSSION

General Classification of the Exact Travelling-Wave Solutions

The Riccati and Extended Auxiliary Equation methods yield the exact travelling-wave solutions which can be systematically classified into three regimes: hyperbolic, periodic, and rational. In this classification, a unified framework is presented for understanding the mathematical structures and the wave behaviours presented by the Caputo time-fractional Kadomtsev-Petviashvili equation.

Wave regime	Riccati formulation	Extended Auxiliary formulation	Main characteristic
Hyperbolic	$(U_{s1}): \tanh, (U_{s3}): \coth$	$(U_{s2}): \tanh, (U_{s3}): \coth$	Smooth bounded and singular hyperbolic travelling structures
Periodic Rational	$(U_{p1}): \tan, (U_{p3}): \cot$	$(U_{p2}): \tan, (U_{p3}): \cot$	Periodic oscillatory structures with singular points
Rational	(U_{r1})	(U_{r2})	Non-periodic inverse-power/algebraic structures

The $\tanh$ and $\coth$ branches are part of the hyperbolic regime. In the former case ($\tanh$ solutions) travelling-wave structures are found which are smooth and bounded and have a finite asymptotic behaviour, while in the latter case ($\coth$ solutions) singular hyperbolic structures are found due to the presence of a pole of the $\coth$ function. Recurrent oscillatory structures are formed within the periodic regime with regularly spaced singular points, the branches of $\tan$ and $\cot$. The rational regime, on the other hand, features an inverse power dependence on the travelling coordinate resulting a non-periodic algebraic behaviour and a singular response at the critical value $\xi = 0$.

The classification shows that the analytical approach is able to describe qualitatively different traveling-wave regimes, but not a single class of solitary structures. Both the Riccati formula and the extended auxiliary formula represent these regimes and the character of the resulting solution is given by the functional branch.

Overall Mathematical and Physical Significance

The classification leads to a direct link between the structure of the auxiliary functions and the qualitative behaviour of the travelling waves. Hyperbolic branches, which are bounded or singular wavy structures, periodic oscillatory regimes with repeated singular manifolds (trigonometric branches), and non-periodic inverse-power algebraic structures (rational branches). So, the analytical solutions offer a comprehensive description of the regular, singular, periodic and algebraically varying wave behaviour in the Caputo time-fractional KP framework.

The comparison which follows is based not only on the explicit forms of the solutions, but also on the mathematical regularity, spatial geometry, singularity structure and physical meaning of the solutions as well as the time modulation due to the fractional order α .

Overall Graphical Interpretation

The solutions obtained by the travelling wave approach are shown in three different graphical representations which are complementary. The 3D surface plots show the geometry of the waves and the overall distribution of amplitude in space in the propagation domain. 2D profiles give cross-sectional pictures of the solutions, so that the shape of the waves, the variation of their amplitude, their regularity and their singular behaviour can be directly assessed along the considered propagation direction. The Contour plots are added to both representations to demonstrate the spatial structure of constant amplitude levels and the periodic repetition and singular manifolds of the $\coth$, $\tan$, $\cot$ and rational branches. Together these representations give a consistent visual representation of the various analytical wave regimes.

Riccati Formulation

The Riccati approach is built by considering an auxiliary equation with a parameter λ which determines the functional class of the found travelling-wave solutions. By its nature, this parameter divides the solution space into three regimes.

For $\lambda < 0$: The auxiliary equation has hyperbolic branches, which results in the *tanh* and *coth* travelling-wave structures. The *tanh* branch has a smooth bounded hyperbolic profile while the *coth* branch has a singular hyperbolic structure due to its pole.

If $\lambda > 0$: The auxiliary equation gives rise to trigonometric branches (tan and cot functions). These solutions are periodic traveling structures that have singular points which repeat in a regular fashion.

If $\lambda = 0$: The Riccati equation can be transformed into its rational form, giving rise to the Inverse Power solution.

A non-periodic algebraic travelling-wave regime $U_{r1}(\xi) = A_0 - \frac{A_1}{\xi} + \frac{A_2}{\xi^2}$.

As a result, the Riccati equation gives a consistent parameter-dependent mechanism of emergence of all the three possible branches, namely, hyperbolic, trigonometric and rational travelling waves, from a common auxiliary structure.

Extended Auxiliary Equation Formulation

The Extended Auxiliary Equation Method is a finite polynomial expansion of the auxiliary function, generally taken as $U(\xi) = A_0 + A_1\psi(\xi) + A_2\psi^2(\xi)$, and the selection of the auxiliary function $\psi(\xi)$ fixes the type of wave Regimes.

Solution Regimes	Method	Functional branch	Parameter regime	Regularity	Periodicity	Singularity	Dominant spatial behaviour	Physical interpretation
Hyperbolic	Riccati Extended Auxiliary		or hyperbolic auxiliary branch	Smooth, bounded	No	No	Smooth hyperbolic transition	Regular localized or front-like travelling structure
Hyperbolic	Riccati Extended Auxiliary		or hyperbolic auxiliary branch	Singular	No	Yes	Pole- dominated hyperbolic structure	Singular hyperbolic travelling structure
Trigonometric	Riccati Extended Auxiliary		or trigonometric auxiliary branch	Singular	Yes	Yes	Repeated oscillatory structure	Singular periodic travelling wave
Trigonometric	Riccati		or	Singular	Yes	Yes	Repeated	Singular
	Extended Auxiliary		trigonometric auxiliary branch				oscillatory structure	periodic travelling wave
Rational	Riccati Extended		Degenerate branch	Singular algebra	No	Yes	Inverse- power	Non-periodic algebraic

	Auxiliary			ic			behaviour	travelling structure
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These hyperbolic choices of the form $\psi(\xi) = \tanh(k\xi)$ and $\psi(\xi) = \coth(k\xi)$ yield smooth bounded and singular hyperbolic travelling structures, respectively. The periodic traveling structures with regularly spaced singular points are given by the trigonometric functions $\psi(\xi) = \tan(\xi)$ and $\psi(\xi) = \cot(\xi)$, respectively, while the singularity that is

rational, $\psi(\xi) = \frac{1}{\xi}$, corresponds to the degenerate auxiliary configuration. The scale parameter k influences the characteristic spatial variation of the function used on the auxiliary, and consequently the width of the hyperbolic structures, the wavelength of the periodic structures and the spatial scaling of the resulting travelling-wave profiles. In general, the formulation of the Extended Auxiliary gives a unified polynomial framework for the construction of the principle solution families of the hyperbolic, trigonometric and rational solution families. [8], [10], [11]

Cross-Regimes Physical Comparison

The solutions of the travelling waves obtained reveal well-separated dynamic and spatial features. Below is a cross- Regimes overview of the key mathematical and physical features of the five functional branches.

Branch	Regularity	Localization	Periodicity	Singularity	Dominant spatial behaviour
<i>Tanh</i>	Smooth, bounded	Finite-transition localized depending on coefficients	No	No	Smooth hyperbolic
<i>Coth</i>	Singular	Concentrated near the pole	No	Yes	Singular hyperbolic
<i>Tan</i>	Singular	Not globally localized	Yes	Yes	Periodic trigonometric
<i>Cot</i>	Singular	Not globally localized	Yes	Yes	Periodic trigonometric
Rational	Singular, algebraic	Algebraically varying	No	Yes	Inverse-power

The difference is that the *tanh* branch gives the regular hyperbolic regime, the *coth* branch gives a singular hyperbolic structure. The *tan* and *cot* branches extend the solution space to a recurrent periodic structure with singular points in between and the rational branch corresponds to an inverse power solution which is not periodic. Thus, the choice of the auxiliary function dictates the qualitative spatial properties of the travelling wave, such as its regularity, periodicity and structure of singularities. This classification is complemented by the three different ways in which the three graphical representations used throughout the analysis exposed the wave behaviour: the 3-D surfaces revealed the global geometry, the 2-D profiles resolved the wave behaviour along a selected propagation direction and the Contour plots revealed singular manifolds and periodic repetition in space. This integrated representation is helpful to differentiate the wave families that can be confused by their similar look in a single model representation.

Fractional-Memory and α -Parameter Analysis

Analysis aspect	Mathematical setting	Effect of varying	Interpretation for the travelling-wave solutions
Classical reference	$\alpha = 1$	Provides the integer-order temporal limit	Serves as the baseline for assessing fractional-memory effects
Fractional regimes	$(\alpha = 0.9, ; 0.8, ; 0.7, ; 0.5)$	Changes the temporal scaling while other parameters remain fixed	Separates memory-induced temporal effects from the spatial structure determined by the auxiliary branch
Temporal propagation	$\xi = \frac{a}{\Gamma} (1 + \beta)x^\beta + \frac{b}{\Gamma} (1 + \gamma)y^\gamma$	Modifies the temporal contribution to and hence the evolution of the travelling structure	acts as a temporal memory/ scaling parameter rather than changing the underlying solution Regimes
Wave displacement	Γ Fixed (t) , varying α	Changes the position of the wave profile or singular manifold through	Fractional memory produces temporal displacement relative to the classical case
Phase evolution	Tan and cot branches: $\tan(k\xi), \cot(k\xi)$	Shifts the temporal phase of the recurring structures	The periodic functional character is preserved, while modulates temporal phase evolution
Hyperbolic-wave evolution	$Tanh$ branch: $\tanh(k\xi)$	Changes the temporal position and evolution of the smooth hyperbolic structure	The functional character is preserved; fractional memory primarily modulates temporal propagation
Singular-structure evolution	$Coth, tan, cot$, and rational branches	Shifts the temporal position and evolution of the corresponding singular manifold through	The singularity is inherent to the analytical branch and is not removed or converted into another functional class solely by varying
Rational-wave evolution	$U_r(\xi) = A_0 \pm \frac{A_1}{\xi} + \frac{A_2}{\xi^2}$	Changes the temporal evolution and location of the algebraic structure	The inverse-power character remains unchanged while fractional memory modulates its temporal progression
Overall interpretation	Auxiliary branch + fractional order α	The branch determines spatial form, while modifies temporal evolution	Auxiliary branch spatial wave class temporal memory and propagation modulation

Physical Interpretation of the Travelling-Wave Regimes and Fractional-Memory Effects

The results illustrate clearly the travelling wave regimes, namely: tanh solutions - smooth, bounded, hyperbolic structures. coth solutions - singular, hyperbolic solutions. tan and cot solutions - periodic structures with repeated singular points. rational solutions - non-periodic inverse-power structure. The fractional order α affects the temporal evolution, displacement, propagation, and phase of these structures via Caputo memory and the auxiliary-function branch defines the spatial wave class underlying them. Therefore, each wave's time course is altered by fractional memory, but without altering its fundamental functional character.

Genuine Methodological and Physical Contribution

The main contributions are the unified and cross-validated analysis of various travelling-wave regimes of the Caputo time-fractional KP equation by means of the Riccati and Extended Auxiliary Equation methods. The parameter correspondence helps to explain related solution branches, and the jointly performed analytical, 3D / 2D / Contour and fractional-memory analyses classify the structures as smooth hyperbolic, singular hyperbolic, periodic singular and rational structures. Thus, it is not the standard tanh, coth, tan or cot, or rational forms, but their systematic comparison, classification, graphical characterization and memory-dependent interpretation in a common Caputo fractional KP framework.

LIMITATION AND FUTURE RESEARCH

There are some limitations in the present study. In the present study, the stability and long-term robustness of travelling wave solutions is not analytically or numerically proved. Furthermore, this study is mostly analytical and graphical construction and characterization of the exact solutions and the sensitivity to change in model parameters and fractional order is not fully quantified. Future investigations will include thorough stability analysis, long-time numerical simulations, parameter sensitivity analysis, and additional numerical studies of these found wave structures to determine their stability, persistence, robustness and sensitivity to changes in the model parameters and fractional order.

CONCLUSION

The Caputo time-fractional Kadomtsev-Petviashvili equation is introduced in this work, by proposing a unified formulation by means of the fractional complex transformation, Riccati and Extended Auxiliary Equation methods. The analytical frameworks lead to the systematic classification of the exact travelling-wave solutions into three main regimes, namely hyperbolic, periodic and rational, with smooth bounded hyperbolic, singular hyperbolic, periodic singular and algebraic rational structures emerging in the three regimes, respectively. Using suitable parameter mappings, the Riccati and Extended Auxiliary Equation methods give solution branches that are consistent with each other, but that help to reveal the mathematical relationship between these two approaches and to differentiate between closely related representations and genuinely different waves. The spatial geometry, regularity, periodicity and singularity structure of the solutions obtained are further characterized by the following three representations: 3D surface, 2D profile and contour. The fractional order α is exactly what introduces temporal memory to the travelling-wave formulation and also modulates the temporal evolution, displacement, propagation and phase of the wave structures without changing its underlying spatial functional class. Therefore, the auxiliary-function branch outlines the main spatial wave nature, while the fractional temporal order is responsible for the temporal realization of that structure, which is dependent upon memory. The novelty of this study is the cross-method derivation of a single family of solutions, the matching of solution parameters, the classification of the various solution families, the description of the different spatial regimes of the solutions in a graphics, and the interpretation of the fractional-memory effects in the same framework for the Caputo fractional KP. The results are mathematically coherent and give a sound mathematical foundation for distinguishing travelling-wave structures in the same fractional model, which are regular, singular, periodic and algebraically varying. The major disadvantage of the present study is that stability and long term robustness of the solutions obtained is not ensured either analytically or numerically. Future studies will then be concentrated on the stability of these found travelling-wave structures, long time numerical simulations and a wider parameter exploration to determine the persistence, robustness and dynamical response of these travelling-wave solutions.

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