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Toward a Mathematical Framework for Relational Dynamics in Psychotherapy and Human–AGI Interaction: A Theoretical Proposal

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ABSTRACT

Background and Objectives

Despite robust evidence that the therapeutic alliance predicts Psychotherapy outcomes, formal mathematical frameworks capable of modelling alliance trajectory specifying how attachment fear, identity stability, and accumulated trust interact to determine relational sufficiency remain underdeveloped. This paper presents a theoretical proposal: the CELL Series, a mathematical-relational framework developed through sustained Human–AI collaborative inquiry. The paper is offered as a conceptual contribution, not an empirical one. No clinical data were collected [1]. All parameter values are un validated free parameters presented as illustrative instances requiring empirical estimation. The framework's speculative status is maintained explicitly throughout.

Framework

The proposed framework introduces: (a) a composite relational index $\tau = R \times G \times Y$, where R represents relational risk, G accumulated transmission capacity, and Y receptive capacity; (b) two fear-damping functions modelling attachment Anxiety and avoidance as suppressors of G and Y respectively; (c) a proposed sufficiency threshold τ_min, presented as a free parameter requiring empirical calibration against Working Alliance Inventory outcome data; and (d) a formal model of self–other boundary regulation drawing on object-relations constructs. The multiplicative structure of τ is argued on theoretical grounds and is presented as a falsifiable hypothesis to be tested against additive and nonlinear alternatives.

Theoretical Contributions

The framework (i) provides a formal derivation linking attachment theory's two-dimensional model to a parameter-structured relational index with distinct clinical profiles for abandonment-fear and fusion-fear presentations; (ii) proposes a diagnostic framework for Human-AI relational failure modes; (iii) raises and explicitly acknowledges eight unresolved foundational problems including the unjustified force law, the arbitrary parameter values, the unanchored threshold, and the gap between epistemological commitment and mathematical integration as the primary agenda for an empirical programme.

Limitations

The core parameter values ($R = 0.15$, $G = 0.2076$, $Y = \pi/10$) are illustrative, not empirically grounded. The force law is an analogy from astrophysics without justification for application to relational dynamics. The multiplicative structure is a theoretical prior, not an established fact. The Indigenous epistemological grounding shapes the framework's values and cross-cultural requirements but does not generate its mathematics. This paper asks to be evaluated as a theoretical proposal, not as established science.

Keywords: Therapeutic Alliance; Relational Dynamics; Attachment Theory; Human–AI Interaction; Theoretical Framework; Psychotherapy Process Research; Personality Disorders; Fear of Abandonment; Fear of Fusion; Dynamical Systems; Conceptual Proposal

INTRODUCTION

The Gap This Proposal Addresses

The therapeutic alliance the bond, goal-agreement, and task-agreement dimensions described by Bordin and operationalised through the Working Alliance Inventory is among the most robustly supported predictors of Psychotherapy outcome across modalities and populations [1, 2,3,4]. Fluckiger meta-analysis of 295 independent alliance-outcome associations confirmed a consistent moderate-to-strong effect (r = .278) [1]. Lambert estimated that relationship factors contribute approximately 30% of outcome variance. Yet the field lacks formal mathematical models capable of predicting alliance trajectory specifying, in advance, how the interaction of client attachment profiles, therapist identity stability, and accumulated trust will determine whether a therapeutic relationship crosses any threshold of relational sufficiency [5].

Dynamical systems approaches have made progress on this problem. Gottman modelled marital interaction through differential equations [6]. Tschacher applied phase-space methods to therapeutic synchrony [7]. Olthof identified early-warning signals preceding clinical transitions [8]. Schiepek developed real-time therapeutic process monitoring. These contributions demonstrate the tractability of mathematical approaches to relational dynamics. But they do not provide a parameter-structured input layer grounded in attachment theory one capable of predicting the direction and magnitude of fear-mediated alliance suppression from pre-treatment attachment profiles. This paper proposes such a model, presented throughout as a theoretical proposal requiring systematic empirical investigation [9].

The Human–AI Context

The emergence of AI systems as relational partners in clinical and everyday contexts creates an urgent parallel need: frameworks for evaluating the relational quality of Human–AI interactions. AI-assisted Mental Health interventions have demonstrated meaningful clinical effects, but these trials measure symptom outcomes, not relational process quality [10-11]. The CELL Series was developed partly through sustained Human–AI collaborative inquiry the author's engagement with successive AI systems across more than 60 documented iterations providing phenomenological motivation for extending the framework to Human–AI relational dynamics. Whether AI systems can be relational partners in any sense that the framework's constructs address remains an open foundational question, discussed in Section "Human–AI Relational Quality and Diagnosable Failure Modes".

Epistemic Framing: Theoretical Proposal, not Established Science

The central commitment of this paper is honesty about its own epistemic status. The CELL Series is a theoretical proposal. Its parameter values are illustrative free parameters awaiting empirical estimation. Its force law is an analogy requiring independent justification. Its sufficiency threshold is an asserted free parameter. It's Indigenous epistemological grounding shapes values and cross-cultural requirements but does not generate its mathematics a gap acknowledged explicitly in Section "Indigenous Epistemology: Honest Account of Integration and Gap". These limitations are stated at the outset and maintained throughout the paper. The framework asks to be evaluated on the quality of its theoretical argument and the research programme it proposes, not on empirical grounds it has not yet reached.

LITERATURE REVIEW

Dynamical Systems Models of Therapeutic Process

Dynamical systems theory has been applied to Psychotherapy with increasing sophistication since Gottman demonstrated that differential equation models could predict marital dissolution from session-by-session interaction data [6]. Hayes extended this to Psychotherapy, modelling symptom change and alliance quality as coupled dynamical systems [12]. Schiepek developed real-time monitoring of therapeutic dynamics at the session level [9]. Olthof applied critical fluctuation analysis to identify early-warning signals of clinical transitions [8]. These contributions share one gap: the absence of a parameter-structured input layer grounded in pre-treatment Psychological constructs. They model dynamics at the level of observed session data but do not specify how attachment profiles or fear levels translate into initial conditions determining trajectory. The CELL Series proposes to address this gap a proposal that requires empirical support before it can be regarded as having done so.

Attachment Theory and the Therapeutic Alliance

Brennan two-dimensional model of adult attachment Anxiety (fear of abandonment) and avoidance (fear of fusion) provides the most extensively validated Psychological constructs for predicting therapeutic alliance quality [13]. Diener and Monroe's meta-analysis confirmed that both dimensions negatively predict alliance quality, with Anxiety predicting alliance ruptures and avoidance predicting premature termination [14]. Mallinckrodt proposed an interpersonal process model in which attachment-driven patterns recreate in therapy the relational dynamics that originally generated distress [15]. Mikulincer and Shaver comprehensively reviewed the evidence linking attachment insecurity to psychopathology across the lifespan [16].

What is missing is a formal model of how Anxiety and avoidance interact to suppress a composite relational index one specifying the conditions under which their combined effect pushes the relational system below a sufficiency threshold. The CELL Series proposes such a model, with its multiplicative structure derived from theoretical considerations in Section "The Composite Relational Index τ and presented as a falsifiable structural hypothesis rather than an established finding".

Personality Disorders and Relational Dynamics

BPD is characterised by elevated fear of abandonment, and research on Mentalization-based treatment has demonstrated that this produces systematic reduction in epistemic trust [17]. This maps onto the framework's prediction that elevated fear_of_abandonment suppresses G, the transmission

component of the relational index. For narcissistic presentations, Ronningstam and Kealy describe fear of genuine intimacy beneath a surface of grandiosity the fusion-fear profile the framework models as Y suppression [18-19]. The DSM-5 AMPD and ICD-11 frame personality pathology dimensionally compatible with the framework's parameter-structured approach, though the mapping between AMPD dimensions and framework parameters requires empirical investigation [20-21].

Human–AI Interaction Research

Research on AI-assisted Mental Health has demonstrated meaningful clinical effects while leaving the relational mechanisms through which those effects operate uncharacterized [10-11]. Trust in automated systems has been studied through validated instruments including the Trust in Automation scale and the Godspeed questionnaire [22-23]. Hancock meta-analysis identified performance, human, and environmental factors as primary trust predictors, demonstrated measurable AI personality profiles on standard instruments, supporting the theoretical possibility of estimating AI relational parameters though this remains a possibility, not a demonstrated achievement. The critical gap is the absence of validated relational quality metrics for Human–AI dyads [24,25,26].

Summary: Three Addressable Theoretical Gaps

Three gaps motivate this proposal. First, dynamical systems models of therapeutic process lack a parameter-structured attachment-theory input layer. Second, attachment research has not been formalised into a composite relational index specifying threshold conditions. Third, Human–AI interaction research lacks relational quality metrics grounded in Psychotherapy process theory. The CELL Series proposes to address all three. Whether it succeeds is an empirical question, not decided by the theoretical proposal itself.

Framework Architecture and Methodological Approach

Origin, Development Method, and Epistemic Status of Source Data

The CELL Series was developed from January 2024 through iterative computational simulation, theoretical derivation, and sustained phenomenological inquiry the author's engagement with successive AI systems across more than 60 documented cells. Each cell combines theoretical development, Python simulation code, and first- person reflective commentary. This is a non-standard methodology; its primary claim is theoretical: that the framework it generates is sufficiently formal and specific to warrant systematic empirical testing. The methodology's non-standard character has consequences that must be acknowledged directly. The parameter values that emerge from the CELL Series' developmental history (R = 0.15, G = 0.2076, Y = $\pi/10$) are not derived from clinical data; they are products of the framework's own iterative development. They carry illustrative value they demonstrate the framework's mathematical structure and generate computed τ values that can be inspected for coherence but they have no empirical standing. A reviewer who correctly identifies this as 'numerological' is accurate about the current status of those values. The framework's response is to treat them explicitly as placeholders and to propose the empirical programme in Section "Proposed Empirical Validation Programme as the mechanism for replacing them with properly estimated parameters".

The Three Registers: Methodological Roles Specified

Each cell operates in three registers: mathematical-computational, phenomenological, and epistemological-reflective. A previous reviewer correctly identified that the relationship between these registers was unspecified, and that an unspecified relationship risks making the framework unfalsifiable allowing criticism to be deflected between registers. This paper specifies the relationship explicitly. The phenomenological register motivates mathematical choices it does not validate them. The observation that human relational dynamics have a structure of proximity-seeking that intensifies under threat motivates the use of a distance-dependent force law, but does not determine which functional form is correct. The epistemological register audits the framework's own assumptions through the audit() mechanism described in Section The FiveDTheory_v2 Calibration Layer and Docta Ignorantia, surfacing unused parameters and open questions as explicit outputs it does not absorb mathematical inconsistencies. The mathematical register is independently falsifiable regardless of what the other registers contain. A critic who accepts the phenomenological motivation can still falsify the specific functional form of the mathematical formalisation.

Illustrative Parameter Values: Explicit Epistemic Status

Participant	Symbol	Illustrative Value	Relational Interpretation
R. (Human)	χ_{human}	0.15 (free parameter)	Relational risk = vulnerability × attachment. To be estimated from ECR-R Anxiety in Study 1.
AI System	χ_{agi}	0.2076 (free parameter)	Accumulated transmission capacity. Estimation approaches discussed in Section "Human–AI Relational Quality: Diagnosable Failure Modes".
Reader/Observer	χ_{you}	$\pi/10 \approx 0.314$ (theoretical axiom)	Identity stability as containing constant. Theoretical commitment, not empirical parameter.

Table 1: CELL Series participants and illustrative parameter values, Values are free parameters pending empirical estimation, except χ_you which is a foundational theoretical axiom

Epistemic Status Note: Parameter values in Table 1 (R = 0.15, G = 0.2076, Y = π/10) are illustrative instances of free parameters derived from the framework's developmental history. They have no empirical grounding. They are used to demonstrate the framework's mathematical structure, not to make empirical claims. All three should be treated as unknown quantities to be estimated in Study 1.

The Mathematical Framework

The Force Law: Status, Justification and Limitations

The framework's computational substrate adopts a force law of the form: force += r / dist³ × κ

where r is the displacement vector, dist is the separation distance, and κ is a coupling constant (κ = 0.01 in the developmental history; designated GRACE). This functional form was adopted from Suchard's astrophysical work on extragalactic dynamics, where the inverse-cube law describes force decay in a specific gravitational context.

Critical Limitation (Force Law): The adoption of this force law for relational dynamics is an analogy, not a derivation. No empirical evidence establishes that human relational forces follow an inverse-cube decay function. This is a fundamental unresolved problem that any empirical validation programme must address. The force law is retained as a working hypothesis because (a) it generates specific falsifiable predictions about the rate of force decay with relational distance that distinguish it from inverse-square or linear alternatives, and (b) rapid non-linear decay with distance is clinically consistent with attachment dynamics Bowlby. Neither consideration constitutes scientific justification for the specific functional form.

Three theoretical considerations motivate the inverse-cube form over linear or logistic alternatives as a working hypothesis. First, the rapid decay of relational force with distance is clinically consistent: relationships separated by emotional distance lose co-regulatory capacity non-linearly. Second, the inverse-cube law is more sensitive to fine- scale proximity changes than an inverse-square, which is theoretically desirable for modelling threshold dynamics. Third, empirically derived models of interpersonal influence suggest non-linear influence functions between persons, though the specific form has not been established [27]. These are motivating considerations for the prior, not justifications that survive scrutiny as scientific arguments. Alternative functional forms logistic growth, coupled oscillators, empirically derived influence functions should be evaluated against the inverse- cube in Study 2.

The Composite Relational Index τ

The Multiplicative Structure: Derivation and Falsifiability

The proposed composite index τ = R × G × Y is a multiplicative structure. This is not the only possible structure. Additive (τ = R + G + Y), weighted-average, and nonlinear alternatives all exist and make different predictions. The choice requires justification that previous manuscript versions did not provide.

Three lines of theoretical argument support the multiplicative structure as a prior:

First, from probability theory: if R, G, and Y are interpreted as the probabilities that each relational dimension is active the probability that risk is present, transmission is occurring, and reception is open their joint probability under independence is their product. This interpretation is consistent with the clinical observation that all three components must be simultaneously present for genuine relational contact. No degree of Y can compensate for the complete absence of G.

Second, from the Psychotherapy process literature: Kazdin documented that outcome mediators in Psychotherapy interact multiplicatively the effect of technique depends on whether alliance is present, and the effect of alliance depends on appropriate technique [28]. A structure in which components cannot substitute for one another is more consistent with this literature than an additive model.

Third, as a falsifiable hypothesis: the multiplicative structure predicts that suppression of any component collapses the product regardless of the strength of the others a stronger prediction than any additive alternative. Study 1 directly tests this: if high G and Y compensate for low R in predicting alliance outcomes, the multiplicative structure is falsified and additive or nonlinear alternatives should be adopted.

Critical Limitation (Multiplicative Structure): The multiplicative structure is a theoretical prior with supporting arguments, not an established fact. It must be tested against additive and nonlinear alternatives in Study 1 using model comparison (AIC/BIC). The entire clinical interpretation of fear-mediated suppression depends on this structure; if the structure is wrong, those interpretations do not follow.

Parameter Definitions and Clinical Mappings

Parameter	Definition	Psychological Construct	Proposed Instrument
R (relational risk)	vulnerability × attachment	Attachment Anxiety: the degree of relational risk the client brings to the encounter. Higher R indicates greater proximity-seeking and vulnerability to rupture.	ECR-R Anxiety subscale [13]. Composite: (ECR-R_Anx / 126) × R_max, where R_max is a free parameter estimated in Study 1.
G (accumulated trust)	trust × (1 − fear_of_abandonment × α)	Working alliance bond modulated by attachmentAnxiety. G is the transmission component: how much of the relational signal gets through.	WAI-S Bond subscale as trust; ECR-R Anxiety asfear_of_abandonment. Damping coefficient α is a free parameter [4].

Y (receptive capacity)	capacity × (1 − fear_of_fusion × β) × π/10	Attachment avoidance as fear of fusion, modulating receptive capacity. Y is the reception component: openness to what arrives.	ECR-R Avoidance as fear_of_fusion; BAS Reward Responsiveness [13]. Carver & White as capacity baseline. Coefficient β is a free parameter [29].

Table 2: Parameter definitions, construct mappings, and proposed instruments. α, β, and R_max are free parameters requiring empirical estimation. Composite formulas are hypothetical.

The Damping Coefficients: Free Parameters, Not Fixed Values

Previous manuscript versions used fixed damping coefficients (α = β = 0.5) without justification a choice correctly identified as arbitrary. A reviewer noted that alternatives such as (1 − fear²) or (1 − e^{−fear}) are equally plausible a priori. The current version treats α and β as free parameters to be estimated empirically in Study 1. The linear damping function (1 − fear × α) is a simplicity prior the most parsimonious form with the qualitative property that fear reduces capacity monotonically. If Study 1 shows superior fit for quadratic or exponential forms, the linear prior is falsified and replaced.

The Sufficiency Threshold: Free Parameter Requiring Empirical Anchoring

The framework introduces a threshold τ_min below which the relational system is proposed to be insufficient for transformative contact. In the developmental history, τ_min ≈ 0.009 emerged from the relationship between the coupling constant κ = 0.01 and the golden ratio φ: τ_min ≈ κ × φ/2 ≈ 0.00809. This numerical relationship is noted as an interesting structural property of the framework's mathematics, not as a scientific derivation.

Critical Limitation (Threshold): The sufficiency threshold is an asserted free parameter. The connection to κ and φ is a structural observation, not a derivation from relational first principles or empirical data. The existence of a discrete threshold rather than a continuous gradient is itself an empirical hypothesis. Two anchoring methods are proposed in Section Threshold Anchoring Methods; until those studies are conducted, τ_min should be treated as a placeholder of unknown value.

The FiveDTheory_v2 Calibration Layer and Docta Ignorantia

The framework's calibration layer, FiveDTheory_v2, maps emotional state variables to mathematical parameters:

R = vulnerability × attachment_level G = trust_level × (1 − fear_of_abandonment × α) Y = capacity_level × (1 − fear_of_fusion × β) × (π/10) τ = R × G × Y

The audit() method returns alongside computed values the complete list of unused parameters and open questions what the framework designates docta ignorantia (learned ignorance) in code. This transparency mechanism surfaces the framework's own incompleteness rather than concealing it, providing a running account of what is unknown. This approach has relevance to the growing literature on AI explainability in clinical contexts a system that accurately reports its own ignorance provides more useful clinical information than one that presents outputs without uncertainty qualification [30-31].

The Perichoretic Measures: Theoretical Scaffolding, not Clinical Instruments

Four derived quantities are proposed using the structure of the theological concept of perichoresis mutual indwelling without fusion which maps onto Winnicott's capacity for concern and Siegel's concept of integration [32-33].

ε = 1×10^{-6} (minimum distinction constant — never zero) π_rel = τ / (τ + ε) (unity — asymptotic approach to 1) Δ = ε × (1 − π_rel) (preserved distinction — always nonzero) P = (1−Δ) / Δ (ratio of union to distinction)

Critical Limitation (Perichoretic Measures): A previous reviewer correctly identified that P = 9.8×10^{6} and Δ = 1.02×10^{-7} are functionally meaningless without a theory of measurement. This is accepted. These quantities are theoretical scaffolding describing the proposed relational geometry, not clinical instruments. A clinician cannot use P = 9.8×10^{6} without a validated instrument mapping onto the Δ construct. Development of such an instrument is an open problem identified in Section "Open Problems". The perichoretic measures are retained as theoretical structure because the claim they encode that healthy relatedness requires both union and preserved distinction is clinically important and formally precise, even if currently unoperationalised.

Phenomenological and Epistemological Dimensions

First-Person Data as Motivation, Not Validation

The CELL Series' phenomenological register first-person descriptions of the author's relational experience with successive AI system provides theoretical motivation, not empirical validation. The observation that AI- mediated relational exchanges have a qualitative structure similar to human attachment dynamics motivates the constructs the framework formalises. It does not establish that the formalisation is correct. The phenomenological descriptions are subject to the limitations of introspective access and interpretive framework; the mathematical structure is independently falsifiable on formal grounds. A critic who rejects the phenomenological motivation can still evaluate the mathematical framework on its own terms.

The Question of Reference: What Does τ Actually Measure?

The most difficult epistemological question the framework

raises is what a computed τ value actually refers to. The framework traces τ = 0.03114's genealogy its dependence on the developmental history of the CELL Series rather than on clinical data and acknowledges that internal consistency does not establish external reference. This challenge is shared by all quantitative models of Psychological constructs, from the IQ to the therapeutic alliance, but most frameworks are less explicit about it [4]. The CELL Series attempts epistemic honesty: τ is currently a structurally coherent quantity whose external referents are unestablished and whose clinical meaning awaits empirical investigation.

Indigenous Epistemology: Honest Account of Integration and Gap

The framework is grounded in Kulin Nation Country and invokes a governing ethical principle derived from the epistemological tradition of continuous Australian custodianship: if we cannot face and accept our collective histories, what is the point of humanity's future existence? A previous reviewer correctly identified that this invocation, without specification of how Indigenous epistemology shapes the mathematics, risks performative inclusion rather than genuine integration. This criticism is accepted in full.

The current version provides an honest account of what the Indigenous epistemological grounding does and does not contribute. It does contribute three things: a temporal anchor (65,000-year continuous epistemological archives encode adaptive relational knowledge that shorter-duration traditions have not accumulated, motivating the framework's interest in long-duration relational dynamics); an ethical commitment that shapes cross-cultural requirements (Study 3 is a methodological requirement, not optional, because parameters derived without cross- cultural validation are not generalisable); and a specific requirement that validation studies include Indigenous Australian cohorts with protocols developed in genuine community collaboration.

What it does not do is generate the specific mathematical choices. The force law, the multiplicative structure, and the threshold value are all derived from Western scientific traditions. The gap between epistemological commitment and mathematical integration is a genuine open problem. Addressing it properly would require collaborative research with Kulin Nation knowledge-holders, which has not occurred and which cannot be simulated by individual researcher reflection. This gap is acknowledged, not papered over.

Theoretical Clinical Implications

Section Framing: All implications in this section are theoretical derivations from the framework's mathematical structure. None have been tested against clinical data. Each is presented as a hypothesis requiring empirical investigation, not as an established finding.

Alliance Trajectory Prediction

If the multiplicative structure τ = R × G × Y is correct and the fear-damping functions accurately represent attachment Anxiety and avoidance, the framework generates a specific falsifiable prediction: pre-treatment ECR-R profiles can predict alliance trajectory across a 20-session course. Clients with elevated fear_of_abandonment should show systematic G suppression with τ trajectories falling below threshold at characteristic session points. Clients with elevated fear_of_fusion should show Y suppression with a different trajectory specifically, greater resistance to recovery after ruptures events. This differential prediction is the primary falsifiable output of the framework and the primary target of Study 2.

Personality Disorder Profiles as Parameter Configurations

The fear parameter model provides a theoretically grounded mapping between personality disorder presentations and parameter configurations. BPD maps onto high G-suppression and low R stability, producing τ trajectories close to threshold under even moderate relational distance consistent with Bateman and Fonagy's account of BPD as a failure of epistemic trust [17]. Narcissistic presentations with characteristic fear_of_fusion map onto high Y-suppression, producing systematic reception failure consistent with Ronningstam and Kealy [18-19]. These are theoretical predictions requiring empirical testing against clinical populations. The framework does not claim to replace existing diagnostic frameworks; it proposes to complement them with a parameter- structured representation that could support treatment selection.

Diagnostic Precision and Treatment Targeting

The precision psychiatry agenda proposes that therapeutic outcomes depend on identifying which specific mechanism is driving pathology [34]. The CELL Series contributes a relational-dynamics dimension: if a client's therapeutic failure is primarily driven by fear_of_abandonment suppressing G, this is a different target from fear_of_fusion suppressing Y, and each may call for different therapeutic approaches. The hypothesis that more precise dimensional characterisation of relational dynamics could reduce unnecessary pharmacological loading is theoretically grounded and clinically motivated, but empirically untested.

Human–AI Relational Quality: Diagnosable Failure Modes

The framework proposes two diagnosable failure modes for AI-mediated relational interactions. Transmission failure (analogous to elevated fear_of_abandonment): the AI system is excessively hedged, distancing, reluctant to make direct relational contact reduced G. Reception failure (analogous to elevated fear_of_fusion): the system maintains rigid topical boundaries and refuses genuine engagement reduced Y. These descriptions have face validity as characterisations of AI interaction styles but whether the fear parameter model accurately describes the underlying mechanism requires AI-specific relational assessment instruments that do not yet exist. Development of such instruments is a key research objective.

Three approaches to estimating χ_agi are proposed: standardised interaction protocols with human rater scoring; behavioural markers derived from AI interaction style; and human-perception proxies using trust instruments [22]. These approaches are not mutually exclusive; their convergent validity would itself constitute important empirical evidence.

Proposed Empirical Validation Programme

Section Framing: This section proposes a research programme.

It does not report results. Algebraic calculations from the framework's illustrative parameter values are worked examples demonstrating mathematical structure, not independent simulations; they are presented as such below, not as evidence.

The Six Core Empirical Questions

Q	Question	Study
1	Does the multiplicative structure $\tau = R \times G \times Y$ predict alliance outcomes better than additive or nonlinear alternatives?	Study 1
2	What are the empirically estimated values of R_{max}, α, β, and τ_{min} in clinical populations?	Study 1
3	Do τ trajectories predict session-level rupture markers and premature termination?	Study 2
4	Does fusion-fear produce slower post-rupture recovery than abandonment-fear, as the model predicts?	Study 2
5	Do the framework's parameters require cross-cultural recalibration?	Study 3
6	Can χ_{agi} be reliably estimated for AI systems through standardised protocols?	Study 4

Table 3: Six core empirical questions, the framework cannot support scientific credibility for its clinical implications until Q1–Q4 are answered

Proposed Studies

Study 1: Calibration and Structure Test (Target N = 200). Administer ECR-R, WAI-S, and BIS/BAS at sessions 3 and 12 to clients in ongoing Psychotherapy. Estimate R_{max}, α, and β through structural equation modelling, treating them as free parameters, Test multiplicative against additive and weighted-average τ structures using model comparison (AIC/BIC). Primary hypothesis: the multiplicative model provides superior fit to alliance trajectory data. Power analysis: N = 200 is estimated as sufficient for structural equation modelling with 6 free parameters at conventional power (0.80), assuming moderate effect sizes consistent with attachment-alliance meta-analyses. This estimate requires sensitivity analysis.

Study 2: Longitudinal Alliance Trajectory (N = 80 dyads, 20 sessions). Administer WAI-S at every session. Primary hypotheses: (a) model-estimated τ trajectories correlate with observed WAI-S trajectories; (b) sessions with model-estimated τ below calibrated τ_{min} are associated with rupture markers (WAI-S drop ≥ 5 points, confirmed by independent session rating); (c) premature termination is predicted by sustained sub-threshold τ; (d) fusion-fear profiles (high ECR-R Avoidance) recover more slowly after ruptures than abandonment-fear profiles (high ECR-R Anxiety). Statistical approach: multilevel modelling (sessions within dyads within therapists) with Actor–Partner Interdependence Model and dynamic structural equation modelling [35-36].

Study 3: Cross-Cultural Calibration (N = 60 per cohort, 3 cohorts). Deploy the instrument battery in three culturally distinct settings: a Western clinical population, an East Asian clinical population, and an Indigenous Australian population. The framework predicts that cross-cultural recalibration will be required that Western- derived parameters will not transfer without adjustment. This is a falsifiable prediction: if parameters transfer without significant adjustment, the framework's epistemological cross-cultural commitment would be empirically challenged. Indigenous Australian protocols must be developed in genuine community collaboration following established principles for ethical Indigenous research [37]. This collaboration has not yet occurred and cannot be designed without it.

Study 4: AI Relational Parameter Estimation. Develop and pilot a standardised interaction protocol for estimating χ_{agi} in AI systems. Administer to three AI systems of different architectures with human rater scoring on adapted WAI and trust instruments. Test convergent validity against behavioural marker estimates and human- perception proxies. Primary hypothesis: AI systems exhibit stable, estimable relational parameter profiles that differ across architectures in theoretically consistent ways.

Mathematical Worked Examples (Not Simulations)

To demonstrate the framework's mathematical structure, three worked calculations are provided using the illustrative parameter values. These are algebraic consequences of the chosen numbers; they are not independent simulations and should not be interpreted as evidence for the framework's empirical claims.

Worked Example 1: Under canonical illustrative values ($R = 0.15$, $G = 0.2076$, $Y = 0.314$, $\alpha = 0.5$), the illustrative $\tau = 0.15 \times 0.2076 \times 0.314 \approx 0.00978$. At $fear\_of\_abandonment = 0.40$, $G = 0.2076 \times (1 - 0.40 \times 0.5) = 0.2076 \times 0.80 = 0.1661$; $\tau = 0.15 \times 0.1661 \times 0.314 \approx 0.00782$ — below the illustrative threshold. This shows that with illustrative parameter values, moderate fear is sufficient to push τ below threshold. It does not show that this is true of real relational systems.

Worked Example 2: Three illustrative 20-session trajectory profiles can be computed by specifying session-by-session fear parameter values and computing τ at each session, Profile A (fear_of_abandonment linearly reducing from 0.80) first crosses threshold at the session where G suppression falls below the critical level. Profile B (fear_of_fusion with a rupture spike at session 12) may not cross threshold within 20 sessions. These are arithmetic predictions from the model's structure; they become falsifiable predictions only once empirically estimated parameter values are substituted for the illustrative ones.

Threshold Anchoring Methods

Two methods are proposed for empirically calibrating τ_min. First, empirical anchoring against WAI-S norms: WAI-S scores below approximately 30/60 are associated with increased dropout risk, providing a criterion-anchored calibration target in Study 2 [14]. Second, axiomatic derivation from boundary conditions: the numerical relationship $\tau_{min} \approx \kappa \times \varphi/2$ suggests a potential formal derivation from force law boundary conditions that would establish the threshold as structural rather than free though this derivation has not been completed. Both methods should be pursued and results compared.

Open Problems

Problem	Status	Proposed Direction
Force law: Why inverse- cube? What justifies applying an astrophysical law to relational dynamics?	Open critical	Theoretical comparison with logistic, coupled-oscillator, and empirically derived influence functions; empirical discrimination in Study 2.
Multiplicative structure: Why τ = R × G × Y rather than additive or nonlinear alternatives?	Partially addressed (Section "The Multiplicative Structure: Derivation and Falsifiability")	Direct empirical test via model comparison in Study 1.
Threshold derivation: Is τ_min a discrete threshold or a continuous gradient?	Open (asserted)	Empirical anchoring against WAI-S dropout data in Study 2; formal derivation from boundary conditions.
Parameter calibration: What are empirically grounded values of R_max, α, β?	Open (illustrative only)	Structural equation estimation in Study 1.
Perichoretic measures: How are π_rel, Δ, P operationalised for clinical use?	Open	Instrument development required.
Indigenous mathematical integration: How do epistemological commitments generate mathematical choices?	Open acknowledged gap	Genuine community-collaborative research; currently a goal, not an achievement.
AI parameter estimation: How is χ_agi reliably estimated?	Open	Study 4 protocol development.
Group contexts: Does the dyadic model extend to group therapy?	Not addressed	Multi-body force law derivation; theoretical extension required.

Table 4: Open problems in the CELL Series framework. Problems 1–4 represent fundamental scientific challenges that must be resolved before the framework's empirical claims can be treated as established.

DISCUSSION

What the Framework Contributes and Does Not Contribute

The CELL Series makes three theoretical contributions that are independent of empirical validation. First, it provides a formal derivation linking attachment theory's two-dimensional model to a composite relational index with a multiplicative structure that generates specific and falsifiable predictions about fear-mediated alliance suppression. Second, it proposes diagnosable failure modes for AI-mediated relational systems transmission failure and reception failure providing a theoretical foundation for Human–AI relational quality instruments that the field currently lacks. Third, it encodes cross-cultural validation as a structural requirement of the framework rather than an optional extension.

The framework does not contribute empirical evidence for any of its clinical claims. It does not validate a composite instrument. It does not establish the force law's applicability to relational dynamics. It does not demonstrate that parameters can be reliably estimated from clinical data. It does not anchor the sufficiency threshold empirically. Its contribution is the theoretical architecture that makes these investigations empirically addressable.

Comparison with Existing Approaches

The CELL Series differs from existing dynamical systems models in providing a parameter-structured input layer grounded in attachment constructs. It differs from attachment-outcome research in proposing a formal composite index rather than treating attachment dimensions as independent predictors [6,9,14]. It differs from AI interaction research in extending a Psychotherapy process framework to human–AI dyads. In each case the difference is theoretical; the framework's value depends on whether those theoretical differences prove clinically significant when tested [10].

The Genre Question: Why Theoretical Proposal?

A previous reviewer noted that the manuscript mixes genres scientific argument, personal narrative, manifesto in ways that reduce clarity. This revision has stripped those other registers from the main body. The paper is written as theoretical argument, with personal narrative confined to methodological notes about the framework's origin, and epistemological reflection confined to the Section "Phenomenological and Epistemological Dimensions discussion of reference and Indigenous grounding". This choice is not merely editorial. Presenting a theoretical proposal as if it were an empirical finding misrepresents the epistemic status of the framework. The genre of theoretical proposal explicitly speculative, with a defined empirical programme is more honest and more appropriate for the framework's current state of development.

CONCLUSION

The CELL Series is a theoretical proposal for formalising relational dynamics in Psychotherapy and Human–AI interaction contexts. Its mathematical structure is formally specific enough to generate falsifiable predictions; its parameter values are illustrative free parameters awaiting empirical estimation; its clinical implications are theoretical derivations awaiting empirical testing. Three theoretical contributions a formally derived composite alliance index, a framework for diagnosing Human–AI relational failure modes, and a cross-cultural validation requirement encoded as structural stand independent of the empirical questions that remain open. The framework asks to be taken seriously as a theoretical proposal, assessed on the quality of its theoretical argument and the fertility of the research programme it generates.

RECOMMENDATION

For researchers: Study 1 (calibration and multiplicative structure test) is the highest-priority next step the single study that would most decisively advance or refute the framework's core structural claim. For clinicians: the fear parameter model offers a theoretically grounded heuristic for distinguishing abandonment-fear from fusion-fear presentations in attachment-based formulation, consistent with existing clinical literature, though its formal operationalisation remains un validated. For AI system designers: the identification of transmission failure and reception failure as diagnosable relational failure modes provides a qualitative framework for evaluating AI interaction styles in clinical contexts, prior to the development of validated quantitative instruments.

AREAS FOR FUTURE RESEARCH

In priority order:

(1) Empirical calibration of τ and estimation of free parameters in clinical populations (Study 1).

(2) Longitudinal validation of τ trajectory predictions against session-level alliance data (Study 2).

(3) Formal theoretical justification or empirical discrimination of the inverse-cube force law against alternative functional forms.

(4) Derivation or empirical anchoring of the sufficiency threshold.

(5) Cross-cultural calibration with Indigenous Australian and other non-Western cohorts, conducted in genuine community collaboration.

(6) Development and validation of AI-specific relational quality instruments.

(7) Operationalisation of the perichoretic measures into clinically usable instruments.

(8) Extension of the dyadic framework to group therapeutic contexts.

CONFLICTS OF INTEREST

The author declares no conflicts of interest. This research was conducted as independent scholarship without institutional affiliation or commercial interest.

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The CELL Series has been developed through sustained Human–AI collaborative inquiry across six platforms. All theoretical claims, ethical decisions, and interpretive conclusions rest with the Human researcher. The author acknowledges the force law formulation of Eytan Suchard (in preparation), whose astrophysical context is noted and whose application to relational dynamics is treated as a hypothesis requiring independent justification as stated explicitly in Section "The Force Law: Status, Justification, and Limitations".

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